
Statistical Methods for Six Sigma

In R&D and Manufacturing

Anand M. Joglekar



A JOHN WILEY & SONS, INC., PUBLICATION

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***To the memory of my parents
and to Jaya, Nikhil, and Neha***

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Preface

Over the past several years, in my attempts to implement statistical methods in industry, I have had the pleasure of working with a variety of people. Some were curious and asked penetrating questions about all the assumptions behind each statistical method. Others did not need the details and were satisfied with the recipe to solve specific design or analysis problems. Still others, at first blush, seemed not to care, but once convinced, became some of the best users of statistical methods. This book owes a lot to all these interactions.

When I first came to America, from IIT, in Mumbai, India, I was fortunate to be an engineering graduate student at the University of Wisconsin, Madison. There was a strong collaboration between the engineering and the statistics departments at Madison, which allowed us to learn how to apply statistical methods to engineering and scientific problems. Later, as an internal and external consultant, I collaborated with a large number of engineers, scientists, managers, and other technical people from a wide variety of industries. I started teaching statistical methods to industry participants. For a number of years, I have used earlier versions of this book to teach statistical methods to thousands of industry participants. The practical problems the participants raised and the questions they asked have helped shape the selection of topics, examples, and the depth and focus of the explanations in this book. I wish to thank the many collaborators and seminar participants.

This book on the applications of statistical methods is specifically