Integral Equations and Iteration Methods in Electromagnetic Scattering

INTEGRAL EQUATIONS AND ITERATION METHODS IN ELECTROMAGNETIC SCATTERING

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Contents

Introduction

In this book we consider the following fudamental items:

- the scattering of electromagnetic waves by inhomogeneous three–dimensional anisotropic structures described in terms of volume singular integral equations;.
- iteration techniques for solving linear operator equations;
- efficient methods of solving integral equations that employ iteration procedures.

The analysis of scattering of electromagnetic waves in inhomogeneous three–dimensional bounded media is extremely important both from the theoretical and practical viewpoints and constitutes the core family of problems in electromagnetics. In the classical setting, it is necessary to find the solution to the Maxwell equations satisfying the radiation condition at infinity and the continuity of the tangential field components on the surfaces where material parameters of the medium undergo breaks. Such a formulation seems rather simple; however, during more than a century researchers have been addressing these problems. The correctness of the setting is one of the most important questions, as well as the solvability and uniqueness.

One of the first in this family was the problem of scattering by a homogeneous dielectric ball [44]. Virtually, this is the only problem that can be solved analytically. The Poynting theorem [14] which was formulated on the basis of analysis of the Maxwell equations with radiation conditions implies the uniqueness of solution for a wide class of the scattering problems. The following statement was used in applied electromagnetics for a long time: if the conditions of the uniqueness theorem are met, then there exists the solution to the corresponding scattering problem. This is true if the operator of the problem is a Fredholm operator in the functional space where the solution is determined. Attempts to prove the Fredholm property using the classical setting were not successful because the radiation conditions were not properly formulated.

For many scattering problems, the existence and uniqueness of solution are proved by reducing to Fredholm integral equations of the second kind. The most complete analysis of such problems is presented in [22] where two families of problems are considered: the scatterer is characterized by (i) a scalar permittivity function which is differentiable everywhere, including the interfaces or (ii) the medium has a constant scalar permittivity (the boundary of the scatterer is a smooth surface).

Many important scattering problems do not belong to these families, for example, in the case of anisotropic structures, media with variable permittivity and permeability that are discontinuous on the boundary, and so on. The difficulty here is as follows. In [22], the scattering problem is reduced to Fredholm integral equations of the second kind, and then the initial problem is examined using the general theory of integral equations. However, within the frames of the general setting, the scattering problems cannot be reduced to classical Fredholm integral equations.

In electromagnetics, volume singular integral equations that are considered with respect to the electromagnetic field vectors and describe the scattering problems in the general formulation have become an object of intense applications since the beginning of 1970s [18]. These equations enable one to simulate inhomogeneous and anisotropic media in a rather simple manner. In addition, they can be used for investigating various practically important scattering problems when the classical setting (a boundary value problem for the Maxwell equations) is either impossible or very difficult. Note, in particular, the scattering in structures with sharp edges, for example, when the body has the form of a homogeneous dielectric parallelepiped. However, these equations are multidimensional integral equations which apparently caused a certain delay in the development of the methods of solution [20,21].

In this study, we perform a detailed analysis of integral equations and corresponding scattering problems, including nonclassical ones, using their general setting. We obtain the necessary and sufficient conditions that provide the fulfillment of the Noether property of the operator and sufficient conditions for the Fredholm property. We prove the existence and uniqueness theorems for the scattering problems considered both in classical and nonclassical settings.

Today, volume singular integral equations are widely applied as an efficient tool of numerical solution to the problems of scattering by complicated three-dimensional structures. In fact, they allow one to overcome difficulties related to taking into account radiation conditions when the Maxwell equations are solved numerically. We describe the methods of constructing efficient algorithms of numerical solution to the considered integral equations. Note that we do not solve particular scattering problems in order to keep open the general idea of the approach.

We pay a considerable attention to iteration techniques, using them to develop computational algorithms. We also employ the minimal residual iteration method to prove the unique solvability for a wide range of scattering problems. We will present the original results of the theory of iteration methods that are of independent significance. We address several iteration methods for linear equations with nonslefadjoint operators acting in functional spaces. (Note that many problems of mathematical physics, including the scattering problems, are nonslefadjoint). The considered iteration methods are well developed for linear equations with slefadjoint operators; in several cases, they apply also to nonslefadjoint operators. We obtain the conditions of convergence that enable one to use these methods for solving linear operator equations.

Let us present a brief survey of the book chapters.

In the first Chapter, we consider iteration methods. The contents is not connected with specific applications to scattering problems and this part of the book may be considered independently. In Section 1, we consider the Jacobi method (JM) with a complex iteration parameter and obtain the necessary and sufficient conditions that provide the convergence of iterations. We also justify the practical way of finding optimal iteration parameters. In Section 2 we study the minimal residual method (MRM). We prove the theorem specifying the conditions that guarantee the convergence. We demonstrate that for many problems of mathematical physics, this condition can be substantiated using the law of conservation of energy. This theorem can be used also for verifying the unique solvability of specific problems. In Section 3 we analyze the multistep MRM (MMRM) that genaralizes MRM and has a much higher rate of convergence. In Section 4, we consider the quasi-minimal residual methods (QMRMs) which employ the idea of biorthogonalization [4,7]. The QMRM convergence is justified and peculiarities of their implementation are discussed. In Section 5 we dwell on numerical methods for solving linear operator equations (for example, integral equations) with the help of iteration techniques and the Galerkin or collocation methods. We demonstrate that the linear equation system obtained as a result of discretization of an operator equation can be solved by using one of the iteration method described in the previous sections. In addition, we prove the convergence of approximate solutions to the exact one as $N \to \infty$, where N in the number of basis functions in the Galerkin method or nodes in the collocation method. In Section 6 we construct an iteration algorithm which enables one to determine the solution of the initial linear equation with the given accuracy using approximate solutions. In Section 7 we compare different iteration methods and describe specific features of their implementation and areas of efficient application.

In the second Chapter we focuse on the problems of scattering of electromagnetic waves in inhomogeneous three–dimensional bounded media. In Section 8 we derive volume singular integral equations that describe scattering problems within the frames of the general setting. We consider the equivalence of the boundary value problems for the Maxwell equations with the radiation conditions and the obtained integral equations and prove the corresponding theorem. In Section 9 we analyze equations in the space L_2 of square-integrable vector-functions. This space is the most appropriate from the physical viewpoint because the Poynting theorem, which constitutes a formulation of the law of conservation of energy, employs integrals involving squared magnitudes of fields. We obtain the necessary and sufficient conditions that provide the fulfillment of the Noether property of the operator and sufficient conditions for the Fredholm property. In Section 10 we prove the existence and uniqueness of classical solution. In Section 11, we study singular integral equations under minimum possible restrictions imposed on material parameters; namely, we assume that they are simulated by bounded functions. We prove the existence and uniqueness theorem for a very wide family of classical and nonclassical scattering problems. In Section 12, we consider the scattering by three–dimensional inhomogeneities situated inside or close to other structures, for example, inside a waveguide or in the presence of perfectly conducting bodies. We prove the existence and uniqueness of solution to corresponding problems.

In the third Chapter we justify the applicability of the considered iteration methods for solving integral equations and propose efficient numerical procedures. In Section 13, we substantiate the JM as applied to the scattering problems. Important details of implementation of the method in the quasistatic wavelength range are presented. In Section 14, we justify the applicability of MRM and QMRM for the numerical solution to the problems of scattering in inhomogeneous anisotropic media. Two types of discretization of the integral equations are considered: the Galerkin method and the collocation. We show that the linear equation system obtained as a result of discretization can be solved by iterations. We also prove that approximate solutions converge to the exact one. In Section 15 we perform a detailed analysis of the algorithms of numerical solution to integral equations employing iteration methods. A regular grid is used to construct efficient procedures of calculating iterations; the number of arithmetic operations per one iteration is estimated by $T \sim N \log_2 N$, where N is the dimension of the linear equation system. We consider peculiarities of numerical implementation depending on physical characteristics of the problem. In Section 16 we consider the scattering of electromagnetic waves by an inhomogeneous three–dimensional anisotropic body in the presence of perfectly conducting surfaces. A system of integral equations is constructed with respect to the electromagnetic field in the domain occupied by the inhomogeneity and the current on the perfectly conducting surface. We justify the applicability of MRM for solving the linear equation system obtained as a result of discretization of integral equations using the Galerkin method.

In Appendix, we present important references to the results of the theory of multidimensional singular integral equations that are used in the second and third chapters.

The results set in this book are published in Refs. [28,30-41].

Chapter 1

Iteration methods for solving linear equations

In this chapter, we consider the following iteration methods for solving linear operator equations: the Jacobi method (JM), which is also called the method simple iteration; minimal residual method (MRM); multistep MRM (MMRM); and quasi-minimal residual methods (QMRM). We will formulate and prove the conditions that provide applicability of the methods. Note in particular that one can apply some results obtained for the MRM to prove the solvability and uniqueness of solution to certain problems. We analyze the schemes of numerical solution to linear equations with the operators acting in functional spaces and compare different iteration techniques.

1. The Jacobi method

In this section, we consider the JM as applied to linear equations with nonselfadjoint operators.

Consider the linear operator equation

$$\mu u - \hat{R}u = f \tag{1.1}$$

with respect to a complex parameter μ. Here u and f are the elements of Banach space E and $\hat{R}$ is a linear continuous operator acting in this space.

From the viewpoint of solvability of equation (1.1), the complex plane of variable μ is divided into two parts: resolvent set $\rho(\hat{R})$ containing all μ, for which equation (1.1) has the unique solution in E for arbitrary right-hand side $f \in E$, and spectrum $\sigma(\hat{R})$ of all the remaining values of μ.

Write equation (1.1) in the equivalent form

$$u - \hat{B}_\chi u = \frac{1}{\mu - \chi} f. \tag{1.2}$$

Here

$$\hat{B}_\chi = \frac{\hat{R} - \chi \hat{I}}{\mu - \chi}, \tag{1.3}$$

is a linear continuous operator, χ is an arbitrary complex number, $\chi \neq \mu$, and $\hat{I}$ is the identity operator in E.

The following statement is known in the theory of linear operators [11].

Theorem 1.1. *Let $\hat{B}$ be a linear continuous operator acting in a Banach space E and*

$$\rho_0 = \sup |\nu'|, \quad \nu' \in \sigma(\hat{B}) \tag{1.4}$$

be the spectral radius. If $\rho_0 < 1$, then the equation $u - \hat{B}u = f$ has the unique solution