

Vaithilingam Jeyakumar and
Alexander Rubinov (Eds.)

CONTINUOUS OPTIMIZATION

**Current Trends and
Modern Applications**

APPLIED OPTIMIZATION

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Applied Optimization

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Panos M. Pardalos
University of Florida, U.S.A.

Donald W. Hearn
University of Florida, U.S.A.

CONTINUOUS OPTIMIZATION

Current Trends and Modern Applications

Edited by

VAITHILINGAM JEYAKUMAR

University of New South Wales, Sydney, Australia

ALEXANDER RUBINOV

University of Ballarat, Ballarat, Australia



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Preface

Continuous optimization is the study of problems in which we wish to optimize (either maximize or minimize) a continuous function (usually of several variables) often subject to a collection of restrictions on these variables. It has its foundation in the development of calculus by Newton and Leibniz in the 17th century. Nowadays, continuous optimization problems are widespread in the mathematical modelling of real world systems for a very broad range of applications.

Solution methods for large multivariable constrained continuous optimization problems using computers began with the work of Dantzig in the late 1940s on the simplex method for linear programming problems. Recent research in continuous optimization has produced a variety of theoretical developments, solution methods and new areas of applications. It is impossible to give a full account of the current trends and modern applications of continuous optimization. It is our intention to present a number of topics in order to show the spectrum of current research activities and the development of numerical methods and applications.

The collection of 16 refereed papers in this book covers a diverse number of topics and provides a good picture of recent research in continuous optimization. The first part of the book presents substantive survey articles in a number of important topic areas of continuous optimization. Most of the papers in the second part present results on the theoretical aspects as well as numerical methods of continuous optimization. The papers in the third part are mainly concerned with applications of continuous optimization.

We feel that this book will be an additional valuable source of information to faculty, students, and researchers who use continuous optimization to model and solve problems. We would like to take the opportunity to thank the authors of the papers, the anonymous referees and the colleagues who have made direct or indirect contributions in the process of writing this book. Finally, we wish to thank Fusheng Bai for preparing the camera-ready version of this book and John Martindale and Robert Saley for their assistance in producing this book.

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Vaithilingam Jeyakumar
Alexander Rubinov

List of Contributors

Adil M. Bagirov

CIAO, School of Information
Technology and Mathematical
Sciences
University of Ballarat
Ballarat, VIC 3353
Australia
a.bagirov@ballarat.edu.au

Selma Belen

School of Mathematics
The University of Adelaide
Adelaide, SA, 5005
Australia
sbelen@ankara.baskent.edu.tr

Gleb Beliakov

School of Information Technology
Deakin University
221 Burwood Hwy, Burwood, 3125
Australia
gleb@deakin.edu.au

Altannar Chinchuluun

Department of Industrial and
Systems Engineering
University of Florida
303 Weil Hall, Gainesville, FL, 32611
USA
altannar@ufl.edu

Nguyen Dinh

Department of Mathematics-
Informatics
Ho Chi Minh City University of
Pedagogy
280 An Duong Vuong St., District 5,
HCM city
Vietnam
ndinh@hcmup.edu.vn

Andrew Eberhard

Department of Mathematics
Royal Melbourne University of
Technology
Melbourne, 3001
Australia
andy.eb@rmit.edu.au

Miguel A. Goberna

Dep. de Estadística e Investigación
Operativa
Universidad de Alicante
Spain
mgoberna@ua.es

Semion Gutman

Department of Mathematics
University of Oklahoma
Norman, OK 73019
USA
sgutman@ou.edu