Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization

The Ideal Risk, Uncertainty, and Performance Measures

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John Wiley & Sons, Inc.

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John Wiley & Sons, Inc.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

ISBN: 978-0-470-05316-4

Printed in the United States of America.

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STR
To my children, Boryana and Vladimir

SVS
To my parents, Veselin and Evgeniya Kolevi, and my brother, Pavel Stoyanov

FJF
To the memory of my parents,
Josephine and Alfonso Fabozzi

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Preface

Modern portfolio theory, as pioneered in the 1950s by Harry Markowitz, is well adopted by the financial community. In spite of the fundamental shortcomings of mean-variance analysis, it remains a basic tool in the industry.

Since the 1990s, significant progress has been made in developing the concept of a risk measure from both a theoretical and a practical viewpoint. This notion has evolved into a materially different form from the original idea behind mean-variance analysis. As a consequence, the distinction between risk and uncertainty, which translates into a distinction between a risk measure and a dispersion measure, offers a new way of looking at the problem of optimal portfolio selection.

As concepts develop, other tools become appropriate to exploring evolved ideas than existing techniques. In applied finance, these tools are being imported from mathematics. That said, we believe that probability metrics, which is a field in probability theory, will turn out to be well-positioned for the study and further development of the quantitative aspects of risk and uncertainty. Going one step further, we make a parallel. In the theory of probability metrics, there exists a concept known as an *ideal probability metric*. This is a quantity best suited for the study of a given approximation problem in probability or stochastic processes. We believe that the ideas behind this concept can be borrowed and applied in the field of asset management to construct an *ideal risk measure* that would be ideal for a given optimal portfolio selection problem.

The development of probability metrics as a branch of probability theory started in the 1950s, even though its basic ideas were used during the first half of the 20th century. Its application to problems is connected with this fundamental question: "Is the proposed stochastic model a satisfactory approximation to the real model and, if so, within what limits?" In finance, we assume a stochastic model for asset return distributions and, in order to estimate portfolio risk, we sample from the fitted distribution. Then we use the generated simulations to evaluate the portfolio positions and, finally, to calculate portfolio risk. In this context, there are two issues arising on two different levels. First, the assumed stochastic model should be close to the empirical data. That is, we need a realistic model in the first place. Second, the generated scenarios should be sufficiently many in order to represent a