

HERVE MOULIN

Cooperative Microeconomics

A Game-Theoretic Introduction



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A GAME-THEORETIC INTRODUCTION

Hervé Moulin

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This book is dedicated to Victor

Contents

<i>Acknowledgements</i>	1
<i>Overview of the Book</i>	3
CHAPTER 1	
The Three Modes of Cooperation:	
Agreements, Decentralization, and Justice	5
1.1. <i>Cooperation in Economic Theory</i>	5
1.2. <i>Cooperation in Political Theory</i>	8
1.3. <i>The Theme of This Book</i>	11
1.4. <i>Direct Agreements: The Efficiency Postulate and the Core</i>	14
1.5. <i>The Justice Mode: End-state Justice</i>	19
1.6. <i>Decentralized Behavior</i>	26
1.7. <i>Procedural Justice</i>	36
CHAPTER 2	
Core and Competitive Equilibrium: One Good and Money	45
2.1. <i>Introduction</i>	45
2.2. <i>The Partial Equilibrium Model</i>	48
2.3. <i>Böhm–Bawerk’s Horse Market</i>	49
2.4. <i>Oligopoly with Binary Demands</i>	52
2.5. <i>Existence of the Competitive Equilibrium under Convex Preferences</i>	59
2.6. <i>Decreasing Marginal Costs: Efficiency</i>	66
2.7. <i>Decreasing Marginal Costs: The Core</i>	73
2.8. <i>Nonconvex Preferences and Empty Cores</i>	77
2.9. <i>Trading Games in the Böhm–Bawerk Market</i>	81
Appendix to Chapter 2	86
A2.1. <i>Proof of Lemma 2.1</i>	86
A2.2. <i>Proof of Lemma 2.2</i>	87
A2.3. <i>Proof of Lemma 2.5</i>	88
Exercises on Chapter 2	91
CHAPTER 3	
Core and Competitive Equilibrium: Multiple Goods	103
3.1. <i>Introduction</i>	103
3.2. <i>House Barter</i>	104

3.3.	<i>The Marriage Market</i>	111
3.4.	<i>Bilateral Assignment</i>	117
3.5.	<i>Assignment Economies</i>	123
3.6.	<i>Arrow–Debreu Economies: Divisible Goods and Convex Preferences</i>	129
3.7.	<i>The Edgeworth Proposition</i>	137
3.8.	<i>Trading Games</i>	139
	Appendix to Chapter 3	146
A3.1.	<i>Proof of Theorem 3.1</i>	146
A3.2.	<i>Proof of Theorem 3.3</i>	147
A3.3.	<i>A Heuristic Argument for the Edgeworth Proposition</i>	149
	Exercises on Chapter 3	150
CHAPTER 4		
	Fair Division: The No Envy Test	163
4.1.	<i>Introduction</i>	163
4.2.	<i>No Envy versus Stand Alone: Two Elementary Examples</i>	168
4.3.	<i>The Fair-Assignment Problem: No Envy Equals CEEI</i>	175
4.4.	<i>The Competitive Equilibrium with Equal Incomes</i>	183
4.5.	<i>Three Examples of the CEEI Solution</i>	188
4.6.	<i>The Egalitarian–Equivalent Solution</i>	195
4.7.	<i>Resource Monotonicity</i>	203
4.8.	<i>Divide and Choose, Moving Knives, and Auctions</i>	205
	Appendix to Chapter 4	213
A4.1.	<i>Proof of Statement (i) in Theorem 4.1</i>	213
A4.2.	<i>The Varian Proposition</i>	214
A4.3.	<i>The Egalitarian–Equivalent Solution in Fair Division with Money</i>	215
	Exercises on Chapter 4	218
CHAPTER 5		
	Fair Division: The Stand Alone Test	239
5.1.	<i>Models of Cooperative Production</i>	239
5.2.	<i>Increasing Marginal Costs: The CEEI Solution</i>	244
5.3.	<i>Increasing Marginal Costs: Stand Alone Test and Egalitarian Equivalence</i>	254
5.4.	<i>Decreasing Marginal Costs: The Stand Alone Core</i>	261
5.5.	<i>Decreasing Marginal Costs: Deterministic Solutions</i>	270
5.6.	<i>Public Goods: The Stand Alone Core</i>	277
5.7.	<i>Public Goods: The Ratio Equilibrium</i>	286
5.8.	<i>Public Goods: Two Egalitarian–Equivalent Solutions</i>	293

5.9. <i>Public Bads and Other Forms of Externalities</i>	301
Exercises on Chapter 5	305
CHAPTER 6	
Production Externality Games	324
6.1. <i>Introduction</i>	324
6.2. <i>Voting Over a Public Good: Majority versus Unanimity</i>	328
6.3. <i>Voluntary Contribution to a Public Good</i>	339
6.4. <i>The Average-Return Mechanism: The Tragedy of the Commons</i>	349
6.5. <i>The Average-Cost Mechanism: A Lesser Tragedy</i>	357
6.6. <i>Serial Cost- (or Output-) Sharing: Improving upon Voting</i>	364
6.7. <i>Serial Cost-Sharing of Partially Excludable Public Goods</i>	377
Appendix to Chapter 6	381
A6.1. <i>Strategy-Proof Voting in the Single-Peaked Context</i>	381
A6.2. <i>The Gibbard–Satterthwaite Theorem</i>	383
A6.3. <i>Strategy-Proof Voting and Condorcet Winners: The Case of Multiple Public Goods</i>	383
Exercises on Chapter 6	387
CHAPTER 7	
Cooperative Games	402
7.1. <i>Games in Characteristic Function Form</i>	402
7.2. <i>The Core: Definition</i>	403
7.3. <i>Universally Stable Families of Coalitions</i>	406
7.4. <i>Convex (Supermodular) Games</i>	408
7.5. <i>Balanced Games</i>	412
7.6. <i>The Shapley Value: Definition</i>	417
7.7. <i>The Shapley Value and the Core</i>	424
Appendix to Chapter 7	428
<i>Proof of Theorem 7.2</i>	428
Exercises on Chapter 7	430
<i>Bibliography</i>	441
<i>Index</i>	451

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Overview of the Book

IN THE PAST fifty years, our understanding of the logic of cooperative processes has been revolutionized by the formal tools of microeconomics, in particular the (neo-classical) competitive analysis and the theory of games.

This book gives a partial account of these developments organized in a systematic conceptual framework. I submit that cooperation between selfish economic agents can be conceived in three fundamental “modes,” namely, direct agreement, justice, and decentralized behavior. I argue that actual cooperative institutions must accommodate all three modes and postulate that an ideal institution is one where all three modes “converge” on the same outcome.

The first mode of cooperation is by *direct agreements* of the concerned agents. This mode is free of any institutional context, as agents engage voluntarily and freely in face-to-face transactions. Under the efficiency postulate, the formal concept of the core describes the self-enforcing agreements. A configuration where the core is empty (a not infrequent pattern) is one where cooperation by direct agreements is not workable. Another drawback of the direct agreement mode is the transaction cost: when many agents are involved and/or when many decisions must be made over a long period of time, transaction costs may become so high as to wipe out the benefits from cooperation.

Each one of the two other fundamental modes of cooperation is a solution of this difficulty. In the *justice* mode, the community of agents produces a mechanical formula to divide equitably the cooperative surplus among the concerned agents. The justification of the formula is given by a set of ethically meaningful axiomatic properties. Some difficult choices are called for when several desirable axioms turn out to be logically incompatible. The just outcome is enforced by the indivisible collective authority (akin to Rousseau’s general will). The justice mode is hard to implement when some of the information necessary to compute the equitable outcome is privately held by the individual participants: they may strategically misrepresent this information, and the eventual outcome will not be just.

In the *decentralized* mode, the community chooses to distribute fully the decision power among individual participants. Cooperation then takes place in a game of strategy, and the role of the collective authority is merely to enforce the rules of the game. The actual cooperative

outcome results from the (noncooperative) equilibrium of selfish interests. The decentralized mode may lead to an inefficient strategic equilibrium (as in the Prisoner's Dilemma) or to the absence of any equilibrium.

The design of a cooperative mechanism tries to combine the virtues of all three modes. An ideal mechanism is one such that i) its noncooperative equilibrium should be unique and compelling, ii) the resulting outcome should be just, and iii) this outcome is also stable in the direct agreement mode. Examples of such convergence of the three modes of cooperation are rare; indeed, a recurrent theme of this book is that the existence of an "ideal" mechanism is often a logical impossibility.

Chapter 1 presents the thesis of this book in nontechnical language.

Chapters 2 and 3 look at the exchange and production of commodities in the private property regime. The core (direct agreement mode) and the competitive equilibrium (decentralized mode) are discussed; the celebrated Edgeworth proposition establishing the conditions under which both concepts pick the same outcomes (hence two modes of cooperation converge) is discussed in some detail.

Chapters 4, 5, and 6 look at the distribution and production of (private or public) commodities in the common property regime (when all agents have equal claims on the resources). Chapter 4 discusses the interpretation of fairness as no envy (justice mode), and its relation to competitive equilibrium analysis. Chapter 5 considers the alternative interpretation of fairness as the stand alone test, and its relation to core analysis. Chapter 6 examines decentralized mechanisms for the production of private or public goods, and their relation to the above fairness properties.

Chapter 7 is a concise exposition of cooperative game theory (in the transferable utility context), focusing on core existence results (direct agreement mode) and the Shapley value (justice mode).

The book is elementary and self-contained. No attempt is made at giving complete proofs of the relevant theorems under the weakest assumptions. Instead, the intuition of the general results is developed in the simplest possible numerical examples. Numerous exercises allow the reader to appreciate the capabilities and the limits of the modeling tools.

The book lies at the crossroads of several research fields actively plowed by the literature of the last four decades. Those are competitive equilibrium analysis, cooperative game theory, collective decision (in particular, axiomatic) and fair division, mechanism design, and the implementation problem. Although all of these tightly related topics are discussed at some length, none of them is properly surveyed.

All bibliographical references are gathered at the end of the book, along with a brief list of books of general interest to our topic.