

Artificial Intelligence: Foundations, Theory, and Algorithms

Christian Blum  
Günther R. Raidl

# Hybrid Metaheuristics

Powerful Tools for Optimization

 Springer

# **Artificial Intelligence: Foundations, Theory, and Algorithms**

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*To Gabi and María and our children Júlia,  
Manuela, Marc, and Tobias. Without their  
love and support life would not be the same.*



# Preface

Research in metaheuristics for combinatorial optimization problems has lately experienced a noteworthy shift towards the hybridization of metaheuristics with other techniques for optimization. At the same time, the focus of research has changed from being rather algorithm-oriented to being more problem-oriented. Nowadays the focus is on solving a problem at hand in the best way possible, rather than promoting a certain metaheuristic. This has led to an enormously fruitful cross-fertilization of different areas of optimization, algorithmics, mathematical modeling, operations research, statistics, simulation, and other fields. This cross-fertilization has resulted in a multitude of powerful hybrid algorithms that were obtained by combining components or concepts from different optimization techniques. Hereby, hybridization is not restricted to different variants of metaheuristics but includes, for example, the combination of mathematical programming, dynamic programming, constraint programming or statistical modeling with metaheuristics.

This book tries to cover several prominent hybridization techniques that have proven to be successful on a large variety of applications as well as some newer but highly promising strategies.

A first introductory chapter reviews basic principles of local search, prominent metaheuristics as well as tree search, dynamic programming, mixed integer linear programming, and constraint programming for combinatorial optimization purposes. The following chapters then present in detail five generally applicable hybridization strategies, including exemplary case studies on selected problems. These five approaches are:

- incomplete solution representations and decoders
- problem instance reduction
- large neighborhood search
- parallel non-independent construction of solutions within metaheuristics
- hybridization based on complete solution archives



While these strategies cover many or even most of the hybridization approaches used nowadays, there also exist several others. The last chapter therefore gives a brief overview on some further, prominent concepts and concludes the book.

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<sup>1</sup> <http://rdlab.lsi.upc.edu>



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