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Alexander Reiter

# Time-Optimal Trajectory Planning for Redundant Robots

Joint Space Decomposition  
for Redundancy Resolution  
in Non-Linear Optimization

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# Time-Optimal Trajectory Planning for Redundant Robots

Joint Space Decomposition for  
Redundancy Resolution in  
Non-Linear Optimization

With a Preface by  
Univ.-Prof. Dr.-Ing. habil. Andreas Müller

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# Foreword

Industrial robotics must address the changing needs of future production systems. Improving productivity and flexibility will remain the major challenge for prospective production systems with reduced energy consumption. The enabling key factors to achieve these goals are novel robotic design principles combined with advanced control concepts. An innovative design principle that is being introduced for industrial robots is the kinematic redundancy, i.e. to use a robotic manipulator that has more degrees of freedom than required to accomplish the intended manipulation tasks. Advanced control concepts shall ensure that a given manipulation task is accomplished efficiently. Time-optimal control schemes are such advanced concepts that allow for task execution in shortest possible time. A seamless combination of these innovative concepts – kinematic redundancy and time-optimal control – does not yet exist.

This master thesis addresses exactly this problem. It presents an original approach to the redundancy resolution that facilitates the numerical solution of the time-optimal control problem. The basis is a non-linear dynamical model for the serial robotic manipulator. Emphasis is always given to a generally applicable approach to the dynamics modeling that allows application of the results to other robotic systems. The presented approach is applicable to any robotic manipulator with a single degree of redundancy, i.e. such that have one more degree of freedom as the task space. The results reported in this thesis represent a progress beyond the state of the art. The reader will get introduced to the basic concepts and to the specific modeling approach.

Optimal control is one of the main research directions of the Institute of Robotics at the Johannes Kepler University Linz, and this master thesis an excellent example for the holistic approach pursued at the institute. In various applications, non-linear, model-predictive, and flatness-based control are also used to derive tailored problem specific solutions. The mathematical basis is always a non-linear dynamical model. This is in particular important for the control of elastic robotic systems. The latter is a research topic that is becoming increasingly important with the advent of light-weight robots. Mobile robotic platforms and humanoids are other topics at the institute that build upon the mathematical modeling.

Andreas Müller  
Head of the Institute of Robotics  
Johannes Kepler University Linz

# Abstract

Industrial robotics applications such as pick-and-place tasks where rapid motions are required, or robot motions along complex paths such as in gluing or laser cutting, have increasingly adopted the use of kinematically redundant serial robots in the last years. Reasons for this can be found in the remarkable characteristics of redundant robots such as the enhanced ability to adapt to the workspace structure or to avoid obstacles by changing the joint configuration of the robot without any end-effector motion. In many cases it is preferred to perform tasks in the shortest possible time leading to time-optimal trajectory planning, the problem of finding trajectories with minimum end times. While solution procedures are readily available for non-redundant manipulators, the challenge of exploiting a robot's kinematic redundancy for minimum-time trajectory planning is not sufficiently covered yet. The present thesis introduces an approach for minimum-time trajectory planning based on a separation method known from literature leading to trajectories that explicitly take advantage of the kinematic redundancy of a manipulator and respect technological and physical constraints of the system. Simulation results demonstrate that the method is applicable to robots of different redundant kinematics and yields time-optimal trajectories.

This work has been partially supported by the Austrian COMET-K2 programm of the Linz Center of Mechatronics (LCM), and was funded by the Austrian federal government and the federal state of Upper Austria.



# Kurzfassung

Bei industriellen Anwendungen wie etwa Pick & Place-Aufgaben, bei denen rasche Bewegungen gefordert sind, oder Bewegungsaufgaben entlang von komplexen geometrischen Pfaden wie beim Kleben oder Laserschneiden, werden in den letzten Jahren vermehrt kinematisch redundante, serielle Roboter eingesetzt. Die Gründe dafür sind in den bemerkenswerten Eigenschaften redundanter Roboter zu finden. Dazu zählen die hervorragende Anpassbarkeit der Roboterpose an strukturierte Arbeitsumgebungen und die Fähigkeit Ausweichbewegungen auszuführen, bei denen Robotergetriebe ohne Veränderung der Position und Orientierung des Endeffektors bewegt werden. In vielen Fällen ist es gewünscht, Bewegungsaufgaben in möglichst kurzer Zeit auszuführen. Dies führt zum Problem der Trajektorienplanung mit optimaler, das heißt in diesem Fall minimaler, Endzeit. Für nichtredundante Manipulatoren ist diese Thematik bereits weitgehend untersucht während für redundante Systeme viele Problemstellungen ungelöst sind. Die vorliegende Arbeit stellt eine Methode vor, die auf Grundlage eines aus der Literatur bekannten Separationsansatzes die kinematische Redundanz eines seriellen Roboters explizit ausnutzt. Dabei können sowohl technologische, als auch physikalische Beschränkungen des Manipulators berücksichtigt werden. Mittels Simulation wird gezeigt, dass dieses Verfahren für kinematisch redundante serielle Roboter verschiedener Bauarten zeitoptimale Trajektorien liefert.

Diese Arbeit wurde im Rahmen des LCM (Linz Center of Mechatronics) nach dem Kompetenzzentren-Programm K2 durchgeführt und mit Mitteln des Bundes Österreich und des Landes Oberösterreich gefördert.

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