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Annals of the International Society of Dynamic Games

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Numerical Methods, Algorithms, and Applications to Ecology and Economics

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Preface

The theory of dynamic games continues to evolve, and one purpose of this volume is to report a number of recent theoretical advances in the field, which are covered in Parts I, II and IV. Another aim of this work is to present some new applications of dynamic games in various areas, including pursuit-evasion games (Part III), ecology (Part IV), and economics (Part V). The volume concludes with a number of contributions in the field of numerical methods and algorithms in dynamic games (Part VI).

With a single exception, the contributions of this volume are outgrowths of talks that were presented at the Eleventh International Symposium on Dynamic Games and Applications, held in Tucson, Arizona, USA, in December 2004, and organized by the International Society of Dynamic Games. The symposium was co-sponsored by the University of Arizona, College of Engineering and Aerospace and Mechanical Engineering, as well as GERAD, Montréal, Canada, and the ISDG Organizing Society.

The volume contains thirty-five chapters that have been peer-reviewed according to the standards of international journals in game theory and applications.

Part I deals with the *theory of dynamic games* and contains six chapters.

Cardaliaguet, Quincampoix, and Saint-Pierre provide a survey of the state-of-the-art of the use of viability theory in the formulation and analysis of differential games, in particular zero-sum games. An important result of viability theory is that many zero-sum differential games can be formulated as viability problems. The main achievements of viability theory are assessed and a number of recent developments are explained, for instance, the formulation of viability problems for hybrid differential games. The chapter contains a substantial list of references.

Chikrii, Matychyn, and Chikrii are concerned with differential games of pursuit and evasion in which one or more players can use impulse controls. The state dynamics are ordinary differential equations that are affected by jumps in the state at discrete instants of time. The method of "resolving functions" provides a general framework for the analysis of such problems and essentially employs the theory of set-valued mappings.

Cardaliaguet investigates a nonzero-sum differential game played by tow players on a line. The dynamics are very simple and players wish to maximize their respective terminal payoffs. In the zero-sum case, the situation is well understood. However, the situation in the nonzero-sum game is completely different. The feedback equilibrium payoffs (FEP) are extremely unstable and small perturbations of the terminal payoffs lead, in general, to large changes in the FEPs.

Ganebny, Kumkov, Patsko, and Pyatko suggest a method for constructing robust feedback controllers for differential games that have linear dynamics with disturbances. The method is based on results from the theory of differential games with geometric constraints on players' controls. The authors also provide an algorithm for constructing a robust control and present simulation results for the practical case of lateral motion control of an aircraft during landing under wind disturbances.

Beck and Filar question a standard assumption of game theory that payoffs in noncooperative matrix games are determined directly by the players' choice of strategies. In real life, palyers may—for several reasons—be unable to execute their chosen strategies. Such inability is referred to as "incompetence." A method for analyzing incompetence in matrix games is suggested, assessed, and demonstrated. The method is motivated by applications where investments in efforts that will decrease incompetence can be made.

Margiocco and Pusillo study the classical Stackelberg game in which the first player is the leader and the second player is the follower. By well-posedness it is meant that the solution exists, is unique, and the approach of "maximizing sequences" is valid. Various general characterizations of Stackelberg well-posedness are provided. Furthermore, hierarchical potential games are considered, and it is proved that some properties of well-posedness are equivalent to the Tikhonov well-posedness of a maximum problem of the potential function.

Part II contains two chapters dealing with the theory of *stochastic differential games.*

Alvaréz and Bardi propose and study a notion of ergodicity for deterministic, zero-sum differential games that extends the one in classical ergodic control theory of systems with two opponent controllers. The connections to the existence of a constant and uniform long-time limit of the value of finite horizon games are established. Moreover, a series of conditions for ergodicity are stated and some extensions to stochastic differential games are provided.

Yeung, Petrosyan, and Yeung address the problem of designing mechanisms that guarantee subgame consistency in the framework of cooperative stochastic differential games with white noise. Recent results of the authors for the case of transferable payoffs (utility) are extended to the—highly intractable—case where payoff cannot be transferred among players.

Part III is devoted to *pursuit-evasion games* and contains four chapters.

Melikyan studies the geometry of pursuit-evasion games on 2-D manifolds. The analysis is done for a variety of game spaces. Because of their simple motion, optimal trajectories are, in general, geodesic lines of the game space manifolds. In some cases there is a singular surface consisting trajectories that are envelopes of family of geodesics. Necessary and sufficient conditions for such singularities are stated. The analysis is based upon viscosity solutions to the Isaacs equations, variational calculus, and geometrical methods.

Shinar, Glizer, and Turetsky propose a class of pursuit-evasion differential games in which two finite sets of possible dynamics of the pursuer and the evader, respectively, are give. These sets are known by both players. The evader chooses her dynamics once before the game starts, and this choice is unobserved by the pursuer. The latter can change his dynamics a finite number of times during the course of the game. Optimal strategies of the players are characterized, and the existence of a saddle point is established.

Crück, Quincampoix, and Saint-Pierre are concerned with pursuit-evasion games with impulsive dynamics (see also Chikrii, Matychyn, and Chikrii in Part I). The system controlled by a player consists of an ordinary differential equation, describing continuous evolution, and a discrete equation that accounts for jumps in the state. For qualitative games, a geometric characterization of the victory (capture) domains are given. For quantitative games, value functions are determined using the Isaacs partial differential inequalities.

Shevchenko studies a game with simple motion in which a pursuer and coalition of evaders move with constant speeds in a plane. The pursuer wishes to minimize the distance to the coalition (defined in a particular way) and terminates the game when distance reduction no longer is guaranteed. The game with two evaders,which can be called a game of alternative pursuit, is studies in detail.

Part IV is devoted to *evolutionary game theory* and *applications*. It contains seven chapters.

Garay examines a situation in which a resident population of interacting individuals is described by a logistic model in which interaction parameters depend on the phenotypes of the individuals. A new mutant clone arises in the population. Among the questions addressed are: what kind of mutuant can (cannot) invade the population, and, if invasion occurs, when does stable co-existence arise? The work establishes a connection between adaptive dynamics and dynamic evolutionary stability.

Mitchell presents an analysis of a resource-consumer model in which individuals are allowed to adaptively vary their resource use as a function of competitor density and strategy. It is demonstrated that habitat specialization, stable minima, community invasibility, and sympatric speciation are more likely when individuals are more efficient at converting resources into viable offspring. The work suggests possible links between species diversity and factors influencing resource conversion efficiency (climate, habitat fragmentation, environmental toxins).

Vincent and Vincent suggest a series of modifications of a well-known model for coexistence. Their starting point is a classical version of the Lotka-Volterra competition equation, which subsequently is made frequency dependent in three different ways and allows the modeling of relative abundance. The purpose is to examine the conditions that determine the relative abundance of species which are in an evolutionary stable state. It is assumed that the ecosystem is at or near an evolutionary equilibrium, and the authors seek evolutionary stable strategies to identify a coalition of individual species.

Hamelin, Bernhard, Nain, and *Wajnberg*, and *Hamelin, Bernhard, Shaiju, and Wajnberg* are concerned with the optimal behavior of foragers that reach a patch at random arrival times. In the first chapter, competition is limited to sharing a common resource. In this case, optimal behavior can be characterized by using a Charnov rule with "carefully chosen" parameters. The second chapter deals with the case of interference competition. Here, an earlier result in the literature is extended to asynchronous arrivals. The resulting problem requires the solution of a war of attrition game with random terminal time. In both chapters, the analysis is valid no matter the arrival law, provided that it is Markovian.

Rael, Vincent, Costantino, and Cushing explore the persistence of corn oil sensitivity in a population of one particular flour beetle. The authors use evolutionary game theory to model and analyze population dynamics and changes in the mean strategy of a population over time. Corn oil sensitivity is a strategy of the flour beetle and is a trait in an evolutionary game that affects the fitness of the organisms. Equilibrium allele frequencies resulting from the game are evolutionary stable strategies and compare favorably with those obtained from experimental data.

Whelan, Brown, and Moll propose a game of resource competition (see also the chapters by Hamelin, et al.). To a forager, food items have three properties that relate to the value of a particular strategy: profitability, richness, and ease of digestion. When foraging on foods that differ with respect to these properties, adjustment (modulation) of gut size and throughput rate leads to a specialization of the digestive system. Modulation of digestive physiology to a particular food type causes different food types to be antagonistic resources. Adjustment of gut volume and processing may promote niche diversification and, in turn, sympatric speciation.

Part V contains ten chapters dealing with the *application of dynamic game theory* to various branches of *economics*.

Beard and McDonald examine the issue of improving the efficiency of water usage. One particular instrument here is the trading of water rights. An important problem is how to design an allocation system for a long period of time such that the desirable properties of the system are sustained at each point of time.

This involves the question of time consistency of the water trading contract (see also the chapter by Yeung, et al. in Part II). A model of dynamic recontracting of water rights is developed and its time consistency properties are assessed.

Alpcan and Basşar investigate a (hybrid) noncooperative game that is motivated by the practical problem of joint power control and base station assignment in code division, multiple access wireless a data networks. Each mobile user's feasible actions include not only the transmission power level, but also the choice of base station. Existence and uniqueness of pure strategy Nash equilibria of the hybrid game are studied. Because of a lack of analytical tractability of the game, numerical analyses and simulations are conducted.

Shen and Başar study pricing issues in communication networks, in particular, the Internet. Pricing is an instrument that can be used to control congestion and prompt efficient network utilization, as well as recover costs of the network service provider and generate revenue for future network development and expansion. A Stackelberg game model with nonlinear pricing strategies is analyzed in which the Internet service provider's pricing policy is designed as an incentive strategy. The problem is studied under complete and incomplete information. In both cases, the game is not, in general, incentive controllable.

Behrens, Caulkins, Feichtinger, and Tragler present a differential game model of international terrorism. A Stackelberg game model is set up (see also the chapter by Shen and Başar). Quantative as well as qualitative analyses are carried out of the incentive strategies of the the two players, called "The West" and an "International terrorist organization." A series of implications of equilibrium behavior are identified, for instance, that outcomes can be considerably improved for "The West" if the game is played for shorter periods of time, or if the terrorist organization's ability to recruit new terrorists can be diminished.

Cellini and Lambertini use a differential game model that represents a variation on the classical Ramsey capital accumulation problem. The game is noncooperative and played by oligopolistic firms. An important aim of the authors is to assess the effects of market power on the long-run equilibrium performance of the economic system. Nonlinear market demand functions are employed, and it is shown that the game admits multiple steady-state equilibria. Also the steady-state relationship between demand curvature and the capital commitments of the players is identified.

Luckraz studies the effects of process imitation on economic growth under stochastic intra-industry spillovers. A main result is that in an economy where the representative industry is a duopoly, R&D spillovers positively affect the economic growth. The game is played in two stages: in the first, the duopolists use Markovian

R&D investment strategies. In the second stage, the duopolists compete in the product market. It turns out that in steady state there is a positive and monotonic relationship between the spillover rate and economic growth.

Kim and El Ouardighi develop a differential game model of collaboration between a manufacturer and a supplier. For each player, the issue is how to allocate her resources among two activities, viz., improving the existing product and developing a new one. In a noncooperative setting, equilibrium time paths of individual player's strategies are identified and confronted with those that would apply in a cooperative game. Numerical simulations suggest that if players cooperate, they prefer to increase the quality of the existing product.

Rubel and Zaccour consider a supply chain (see also the chapter by Kim and El Ourdighi) with one manufacturer and one retailer. The former can choose to sell directly to the consumers (e.g., by e-trade). A noncooperative differential game model is suggested in which players control their respective marketing efforts that are directed at keeping/attracting consumers to a player's preferred channel (i.e., the conventional one for the retailer, the online one for the manufacturer). The authors identify a feedback Nash equilibrium in an infinite-horizon,linear-quadratic game.

Jørgensen and Di Liddo suggest a two-stage game of fashion imitation. A fashion firm sells its product in an exclusive, high-price market for a limited period of time. Afterwards, a less expensive version of the product is introduced in a market with many more consumers. In this market, the fashion firm faces competition from a fringe of firms that sell an illegal imitation of the fashion firm's product. One problem for the fashion firm is that its brand image is diluted by excessive amounts of goods sold in the lower-price market (including its own).

Friesz, Mookherjee, and Rigdon wish to advance the use of differential variational inequalities (DVI) in noncooperative Cournot–Nash dynamic games. With this purpose in mind, the chapter provides a guided tour of how to use DVI in formulating, analyzing, and computing solutions in such games, As a practical example, the author consider a problem of competitive revenue management that is related to pricing and resource allocation in service networks.

Part VI contains six chapters that deal with the development and use of *numerical methods and algorithms* in dynamics games.

Kushner considers a class of stochastic zero-sum differential games with a reflect diffusion system model and ergodic cost criterion. The controls of the players are separated in the dynamics as well as in the cost function. Such ergodic and "Separated" models sometimes occur in risk-sensitive and robust control problems. The numerical method proposed by the author solves a stochastic game for a finite-state Markov chain and an ergodic cost criterion. Essential conditions are nondegeneracy of the diffusion and a weak local consistency condition.