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Fast Simulation of Electro-Thermal MEMS

Efficient Dynamic
Compact Models

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MICROTECHNOLOGY AND MEMS

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With 108 Figures, 8 in Color and 9 Tables

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Dedication

TB: For Dennis

Preface

In 2001, an electrical engineer, a chemist and a mechanical engineer needed to solve a common problem: to develop a compact electro-thermal model for system-level simulation. An initial idea was to use the conventional route of compact thermal modeling but fortunately we found a much better alternative: formal model order reduction. This book presents results of our research and an introduction to both electro-thermal simulation and model order reduction.

We are practitioners and the book is written for practitioners as well. A background in mathematics is required but the main emphasis of the book is on the original ideas. There are many practical examples that show how one can use model reduction to build an accurate dynamic compact model in an automatic fashion directly from a finite element model.

When we started this work, there was almost no software to use modern model reduction in practical work. Over time we have developed a software environment that allows us to start from a thermal model developed in ANSYS®, perform model reduction directly for an ANSYS model, and then simulate the reduced model and visualize the results. The software is described in the book and available from our Web site. Even if the background mathematics happens to be too difficult to comprehend, a reader will still be able to perform model reduction of their own models.

Model reduction is quite a general technique and it is not limited to thermal problems. On the other hand, it happens that a thermal problem is the most natural problem for the application of model reduction. As such, the book will be a good introduction to model reduction for any engineer even though its main focus is electro-thermal simulation.

Acknowledgments

The book was written while working in the Simulation Lab of Professor Korvink, and we would like to thank all our colleagues for the nice working atmosphere.

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The authors

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Introduction

This text addresses the application of a modern mathematical approach, referred to as model order reduction (MOR), for efficient simulation of electro-thermal microsystems. It is written for engineers that use high-dimensional finite element models (alternatively, spatial discretization can be used as well) during device simulation. It describes the automatic generation of accurate dynamic compact thermal models directly from finite element models. Such compact models are convertible into hardware description language form and can be directly used in system-level simulation or employed for extremely effective design optimization of electro-thermal microsystems.

In general, the design of microsystem devices often depends on large-scale transient simulation of coupled physical domains, such as thermal, mechanical, electrical, etc. This requires the solution of very large systems of ordinary differential equations (ODEs), resulting from the spatial discretization of a computational domain. However, instead of a common “brute force” approach to integrate a large system of ODEs, one can use modern mathematical methods to drastically reduce the problem dimension, and thereby achieve dramatic speedup of the calculation time. Hence, nowadays it is possible to simulate models that only several years ago were too large (due to lack of time, computer memory or computer speed). Indeed, it has been shown that for many MEMS devices, such as accelerometers, gyroscopes and many different electro-thermal devices, the number of ODEs obtained from finite element modeling can be reduced by several orders of magnitude almost without sacrificing precision.

This book describes a complete MOR methodology and software environment at the engineering level. It is equipped with a large number of practical examples, to show readers how to considerably speed up simulation in a concrete problem. Although the model order reduction approach can be applied to different physical domains, in this book we focus on electro-thermal MEMS.

1.1 MEMS, Compact Modeling and Model Order Reduction

The development of increasingly complex microstructures (in the following we will call all microsystems MEMS¹, even if functionality other than micro-electromechanical is employed) demands sophisticated simulation techniques for design, control and optimization [1, 2]. Often, system-level simulation, which includes several single devices placed on a chip together with their driving circuitry, is indispensable. Although no universal simulation strategy currently exists to cover all MEMS design situations [3], reduction of the problem size drastically reduces the computational work. This book is an overview on how to automatically produce reduced models for system-level simulation of MEMS, using modern mathematical approaches.

Traditionally size reduction is performed via compact modeling, which was developed in electrical engineering long before MEMS. The goal of compact modeling is to create a small size equivalent network of resistors, capacitors, inductors, etc. which accurately describes the dynamics of the device and can be directly inserted into SPICE-like simulators. Naturally, MEMS engineers try to use the same methodology. Mathematically speaking, compact modeling starts by choosing the topology of a small-dimensional equivalent circuit (see Figure 1.1). During the second step, parameters within this network (resistivities and capacitances) are found by fitting model parameters to measured or simulated curves. This approach requires the designer to choose the correct network topology intuitively, i.e. without strict guidelines, and then to perform a model parametrization. It should be noted that although the second step requires time-consuming data fitting, the first step usually takes even more time in practice as it is based on intuition.

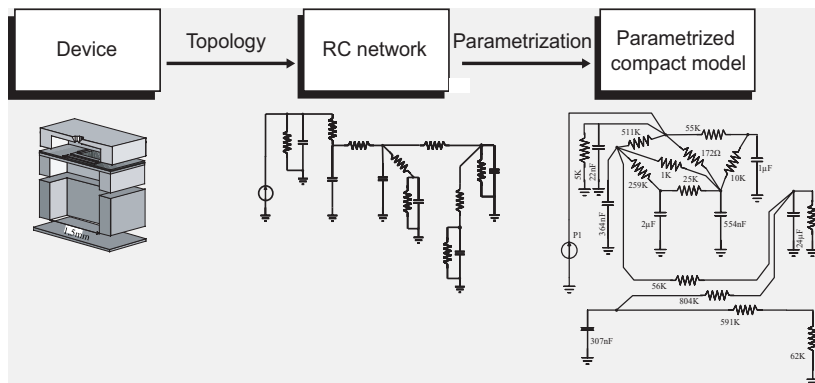


Fig. 1.1. Compact modeling flow of the example thermal MEMS model. RC network pictures courtesy of M. Salleras (UB, Spain).

¹MEMS traditionally stands for micro-electromechanical systems.

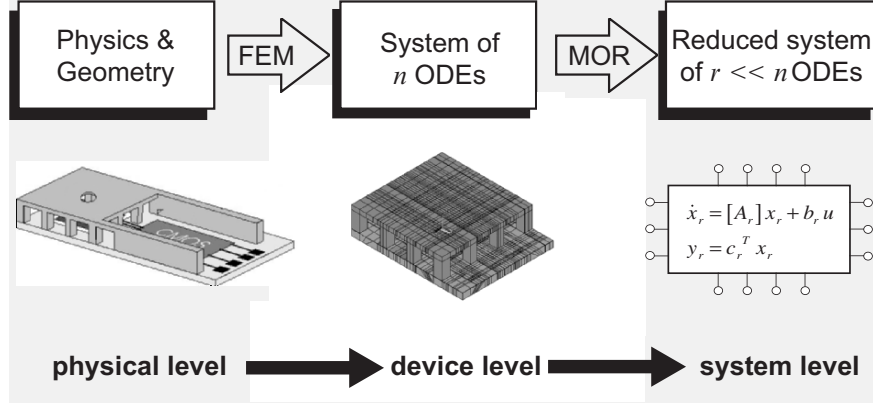


Fig. 1.2. Model order reduction: a switch from device to system level simulation.

An alternative to compact modeling is mathematical model order reduction (also called the approximation of large-scale dynamic systems [4]). Figure 1.2 shows the MOR flow. The simulation of a single device starts with the governing partial differential equations (PDEs). One example is a heat transfer PDE, Eq. 2.5, which takes a central place in this book. The next step is the discretization in space of the original PDE, using, for example, the finite element method (FEM), which integrates the PDE over a number of small nonoverlapping subsets of the complete domain. This results in a system of ordinary differential equations, whose dimension is proportional to the number of introduced nodes. The finer the spatial discretization required, the more nodes are produced. Due to the complex nature of MEMS, its discrete models are usually large (100,000 equations are the engineering standard nowadays). The second step is mathematical model order reduction, which is based on the transformation of a high-dimensional system of ODEs to a low-dimensional one, done by projection. This conversion is formal, robust and can be fully automated. If necessary, the reduced system can also be represented as an equivalent electrical circuit and inserted into a system-level simulator. Hence, mathematical model order reduction can be considered as “compact modeling on demand” [5].

In Table 1.1 the most important properties of compact modeling and mathematical model order reduction are contrasted. Compact modeling requires the designer to intuitively choose the topology of a small-sized equivalent network, which is not a trivial task. It further requires either simulation of the original large-scale system or the use of experimental data. These are then used for the parametrization (via data fitting) of the compact model. No mathematical properties of the original system matrices, or their connection to the matrices of the reduced ODE system are taken into account. Model order reduction, on the other hand, does not require simulation of the original system. Instead, it reduces the original large-scale system matrices using the concept of mathematical projection (explained in detail in Chapter 3). Hence, the reduced system is obtained completely formally and without relying on intuition.