Annals of the International Society of Dynamic Games

# Advances in Dynamic Games

## Applications to Economics, Management Science, Engineering, and Environmental Management

Alain Haurie
Shigeo Muto
Leon A. Petrosjan
T.E.S. Raghavan
Editors

Birkhäuser

Annals of the International Society of Dynamic Games

# Advances in Dynamic Games

## Applications to Economics, Management Science, Engineering, and Environmental Management

Alain Haurie
Shigeo Muto
Leon A. Petrosjan
T.E.S. Raghavan
Editors

Birkhäuser

# Annals of the International Society of Dynamic Games
# Volume 8

Annals of the International Society of Dynamic Games

# Advances in Dynamic Games

*Applications to Economics,*
*Management Science, Engineering,*
*and Environmental Management*

Alain Haurie
Shigeo Muto
Leon A. Petrosjan
T.E.S. Raghavan
*Editors*

Birkhäuser
Boston • Basel • Berlin

Alain Haurie
ORDECSYS
Place de l'Etrier 4
Chêne-Bougeries CH-1224
Switzerland

Shigeo Muto
Tokyo Institute of Technology
Graduate School of Decision Science
and Technology
2-12-1 Ookayama
Tokyo 152-8552
Japan

Leon A. Petrosjan
State University of St. Petersburg
Faculty of Applied Mathematics
and Control Processes
198504 St. Petersburg
Russia

T.E.S. Raghavan
University of Illinois at Chicago
Department of Mathematics, Statistics,
and Computer Science
Chicago, IL 60607-7045
USA

Mathematics Subject Classification: 49N70, 49N75, 49N90, 91-XX, 91Axx, 91A05, 91A06, 91A10, 91A12, 91A13, 91A15, 91A18, 91A20, 91A22, 91A23, 91A24, 91A25, 91A30, 91A35, 91A40, 91A43, 91A44, 91A46, 91A50, 91A55, 91A60, 91A65, 91A60, 91A80

**Library of Congress Control Number:** 2006924926

ISBN-10 0-8176-4500-4 e-ISBN: 0-8176-4501-2
ISBN-13 978-0-8176-4500-7

Printed on acid-free paper.



Printed in the United States of America. (TXQ/EB)

9 8 7 6 5 4 3 2 1

*www.birkhauser.com*

# Contents

# Part VI Management Science Applications

# Preface

This edited volume contains a selection of chapters that are an outgrowth of the 10th International Symposium on Dynamic Games organized by the ISDG, in St. Petersburg, Russia, in July 2002, with a few additional contributed chapters. These fully reviewed chapters present an outlook of the current development of the theory of dynamic games and its applications to various domains, in particular, energy and the environment, economics, and management science.

It is now well established that the paradigms of dynamic games play an important role in the development of multi-agent models in engineering, economics, and management science. The ability of the concepts in providing insight for difficult real-life decision problems stems from their capacity to encompass situations with uncertainty, incomplete information, fluctuating coalition structure, and coupled constraints imposed on the strategies of all the players. The twenty-one chapters grouped in the six parts that constitute this volume cover these different aspects of modern dynamic game theory.

While the foundations of discrete dynamic game theory were laid down by L. Shapley and D. Blackwell, the continuous case in the form of differential games was initiated by R. Isaacs in the USA and by L.S. Pontryagin in Russia. Almost all of their efforts were directed towards the study of zero-sum two-person games. Since the seminal papers by J. Case and A.W. Starr and Yu chi Ho on nonzero-sum, $m$-player differential games, numerous applications to management science, and economics have been presented, particularly in the previous volumes of the Annals of the ISDG series. In this volume several problems pertaining to pursuit-evasion, marketing, finance, climate and environmental economics, resource exploitation, and auditing and tax evasions are addressed using dynamic game models of various sorts. The volume also includes some chapters on cooperative games, which are increasingly drawing dynamic approaches to their classic solutions. The contributions are grouped in six parts.

**Part I** deals with *zero-sum game theory* and contains three chapters.

*Rosenberg, Solan, and Vieille* consider zero-sum stochastic games where players do not observe the progress of the game fully but only in bits and pieces. The key theorem is to show the existence of max-min and min-max for all zero-sum stochastic games with imperfect monitoring which extends an earlier work of Coulomb for imperfect monitoring zero-sum stochastic games with all but one state absorbing. The zero-sum assumption is crucial, as a slight deviation from imperfection for nonzero-sum stochastic games may fail to give any equilibrium payoff.

*Kumkov and Patsko* consider a zero-sum linear differential game with fixed terminal time. Given two sets $A$ and $B$, the set $A \overset{*}{-} B = \{x : B + x \subseteq A\}$. The set $A$ is swept by the set $B$ if $A = B + (A \overset{*}{-} B)$. For any constant $c$, the level set of a real-valued function $f$ is the set $M_c = \{x : f(x) \leq c\}$. The function $f$ is said to have the level sweeping property if for any $c < d$ the set $M_d$ is swept by the set $M_c$. The main result of the work is the proof of the fact that if the payoff function depends on just two components of the phase vector and also possesses the level sweeping property, then so does the value function for the linear differential games. Such an inheritance of the level sweeping property by the value function is specific for the case when the payoff function depends on two components of the phase vector under very general regularity conditions for such differential games. If it depends on three or more components of the phase vector, this inheritance is generally lost. The latter is shown by a counterexample.

*Serov* considers the game of the generalized shortest path problem, where the task is to transit optimally from a fixed point through a system of intermediate sets in $R^d$ to a fixed destination point (or set), with the condition that no point of the sets visited is visited again. The (combinatorial) cost to minimize is assumed additive or bottleneck. It becomes a zero-sum dynamic game when, say, player II decides to choose the order of the sets to visit or to terminate at each stage while player I chooses a point of the set to be visited. For this multistage game problem both open-loop and feedback settings are suggested. The feedback problem is posed in the class of feedback strategies and these strategies can change a route during a motion, depending on current moves of the opponent. They provide, in general, a strictly better value of the problem compared to the open-loop minimax setting. The author shows how to construct an optimal feedback minimax strategy, and some heuristics are also proposed.

**Part II** is concerned with *pursuit-evasion games.* It contains three chapters that address the now classical problems of pursuit-evasion and the related domain of zero-sum differential games.

*Shinar and Glizer* take up the problem of pursuit-evasion where the pursuer's information about the evader's lateral acceleration is delayed. The pursuer needs to estimate this, essentially based on the available measurement history during this delay period. This approach reduces the uncertainty set of the pursuer, due to the estimation delay, by considering in addition to the current (pure feedback) measurements also the available measurement history during the period of the estimation delay. The reduced uncertainty set is computed by solving two auxiliary optimization problems. By using the center of the new uncertainty set's convex hull as a new state variable, the original game is transformed into a nonlinear delayed dynamics game with perfect information for both players. The solution of this new game is obtained in pure strategies for the pursuer and mixed ones for the evader. The value of this game (the guaranteed miss distance) is substantially less than the one obtained in previous works by using only the current measurements.

*Chikrii* extends the well-known Pontryagin sufficiency conditions of capture in ordinary differential games to game problems for systems with fractional derivatives of arbitrary order. These are games with evolution described by equations with fractional derivatives, one for each player. Here player II strives for the state variables to get closer to the state variables of the opponent to within a specified distance. Player I wants them as far away as possible.

*Petrov and Vagin* study the problem of group pursuits and evasion. They provide necessary conditions for the capture of several evaders in a group pursuit problem, where all evaders use the same control. Necessary conditions for capture in such a group pursuit problem are also obtained for a special case called “soft” capture.

**Part III** contains four chapters concerned with *games of coalitions.*

*Petrosjan* introduces the notion of an $n$-person cooperative stochastic game. The Shapley value for cooperative $n$-person transferable utility (TU) games and the value in stationary strategies for zero-sum two person discounted stochastic games are central to the study of cooperative games and dynamic games, respectively. Petrosjan combines these two distinct value concepts: Players in a given coalition $S$ might join together and play the game as though the rest are against them and treat this game as a zero-sum stochastic game. This game has a value and this induced value can be taken to be the worth of the coalition. This in turn determines its Shapley value. The subtree of cooperative trajectories maximizing the sum of expected players' payoffs is defined and the solution of the game along the paths of this tree is investigated. The new notion of cooperative payoff distribution procedure (CPDP) is introduced to show that the resultant Shapley value constructed is time consistent.

*Funaki and Yamato* study the problem of consistency for the core as the solution of a cooperative game with transferable utility. While one can have many reduced games using the same solution, not every one is capable of inheriting the solution for the reduced game. While an earlier characterization used a specific reduced game and used three axioms on this reduced game that characterized the core of a balanced game, here the authors provide a new set of four axioms to characterize the core via a new reduced game.

*Scheffran* proposes a framework for analyzing the interaction between individual players (actors) and collective players (coalitions) who mutually adapt the allocation of investment to their values and each other's decisions. The dynamic process of coalition formation is described by a coupled evolutionary game of allocation controls. An application to analyzing the management of energy and carbon emissions is discussed in some detail.

*Raghavan and Sudhölter* survey the geometric and algorithmic aspects of solutions, like the core and the nucleolus. While a cooperative TU game in general is defined by its characteristic function, some special classes of cooperative TU games are easily determined by a small amount of data. Assignment games belong to this category. They are models of two-sided markets. Players on one side, called sellers, supply exactly one unit of some indivisible good, say, a house in exchange for money, with players from the other side, called buyers. Each buyer has a demand for exactly one house. When a transaction between a seller and a buyer takes place, a certain profit accrues. The worth of any coalition is given by the total profit of an optimal assignment of players within the coalition. Therefore, the characteristic function is fully determined by the profits of the buyer-seller pairs. They study conditions for the core to be a stable set in the sense of von Neumann and Morgenstern. Another solution, called the modiclus, is a spin-off from the notion of nucleolus, taking into account the jealousy between coalitions for any given payoff configuration. Unlike the nucleolus, the modiclus is not in general a core element even when the core exists. However, from a computational point of view, when the modiclus is in the core one has hopes of computing the modiclus efficiently.

**Part IV**, which discusses *new concepts of equilibrium*, is composed of three chapters proposing new interpretations of the interdependence between different members of a social group.

*Vasin* shows that "natural" evolution of behavior in repeated games in human populations is a very unstable process which may be easily manipulated by outside forces. Any feasible and individually rational payoff of the game may be converted into a globally stable outcome by arbitrary small perturbation of the payoff functions in the repeated game. He shows that this result also holds for a trembling-hand perturbation of the game, and proves a new version of the Folk theorem for this case.

*Morgan and Patrone* study the Stackelberg equilibrium, which can be thought of as a subgame perfect equilibium in an extensive game when the leader makes the first move in the leader-follower games. The problems here often stem from the nonuniqueness of the best reply and the differences between weak and strong Stackelberg equilibria. In particular, the authors study Tikhonov regularization methods for seeking the so called weak or strong lower Stackelberg equilibria.

*Hidano and Muto* study the philosophical problem: What triggers two selfish individuals to unite? They treat this as a two-stage decision process, namely to unite or not to unite as the first-stage decision and in case one of the players prefers not to unite, then their aims are simply to maximize their individual utility levels. If they both choose to unite, then they both equally enjoy the maximum utility given by $\max\{h(s,t) \mid (s,t) \in R\}$. The authors discuss subgame perfect equilibria of the game in order to make clear under what conditions different selves unite. Since they have symmetric utility, symmetric equilibria are natural topics to be studied.

In **Part V** four chapters address original *applications to energy/environment economics.*

*Kryazhimskii, Nikonov, and Minullin* develop an explicit algorithm to approximate Nash equilibria for an earlier model of one of the authors on nonzero-sum games of timing for building gas pipelines. In the energy market, say for building gas pipelines, if there are no competitors, then any monopolistic pipeline builder can concentrate on the right time to stop construction and venture into supply that will maximize the rate of return on the initial investments on constructing pipelines. If there are no competitors, then when to start commercialization (stop construction) is often an optimal control problem. But with a competitor it is no longer an optimal control problem. These are games of timing, specifically nonzero-sum games. The Nash equilibria for these games of timing can be approximated. The key point in their approach is based on the observation that the best response commercialization times for all players concentrate at two time points, one corresponding to a fast investment policy and another corresponding to a slow investment policy.

*McKelvey and Golubtsov* study the dynamic fishery harvesting game in a stochastic environment, in order to examine the implications of incomplete and asymmetric information. The main emphasis is on a split-stream version of the game: At the beginning of each harvest season the initial fish stock (or recruitment) divides into two streams, each one accessible to harvest by just one of the two competing fishing fleets. The fleets simultaneously harvest down their streams, achieving net seasonal payoffs for the catch. After harvest, the residual sub-stocks reunite to form the brood-stock for the subsequent generation. The strength of this subsequent generation is determined by a specified stock-recruitment relation, and the cycle repeats. In this cyclic process, both natural environmental factors (stream-split proportions and stock-recruitment relation) and economic factors (harvest costs and benefits) incorporate Markovian stochastic elements. The implications of alternative knowledge structures are explored through dynamic programming and simulation.

*Haurie, Moresino, and Viguier* propose a two-level hierarchical differential game to represent the possible negotiations of GHG emissions quotas among different nations in a post-Kyoto era. The players are countries with growing economies. The quotas are determined noncooperatively but are globally constrained to satisfy a long-term limit on the discounted cumulative emissions. Once the quotas are determined, an international emissions trading system permits the country to realize the abatement program at the least global cost. The set of normalized equilibria is proposed as the solution concept that could be used to drive the negotiations in such a context. These equilibrium solutions are characterized.

*Haurie* proposes a model of intergenerational equity to deal with the long-term issue of climate change. The economic impacts of global climate change are far-reaching for the nations of the world. The myopic gain for one generation has to be balanced with the welfare for future generations. In his paper on a stochastic multigeneration game with application to global climate change, Haurie models this problem as a continuous-time, piecewise deterministic game where the players represent successive generations each with a random life duration. The intergenerational equilibrium concept is applied to a model of integrated assessment of climate change.

**Part VI** contains four chapters that deal with *management science applications.*

*Bernhard, Farouq, and Thiery* investigate a differential game motivated by a problem in mathematical finance, specifically in the theory of option pricing. The nature of one of the players, called the pursuer, is quite impulsive, resulting in unexpected jumps in the state variables. The authors' approach is to rigorously derive the optimal controls which consist of an initial impulse and a long static coasting followed by finitely many controls later. They investigate the convergence problem of an appropriate Isaacs equation for the discrete version and its convergence to the value function via an equivalent but nonimpulsive differential game.

*Jørgensen, Taboubi, and Zaccour* consider the problem of a manufacturer and a retailer allocating their advertisement budgets in national advertising and local advertising, respectively. It is modelled as a Stackelberg game with the manufacturer acting as the leader and the retailer as the follower. The manu-