OPT ART

From Mathematical Optimization to Visual Design

Robert Bosch

Opt Art

OPT

Robert Bosch

ART

From Mathematical Optimization to Visual Design

PRINCETON UNIVERSITY PRESS
Princeton and Oxford

Published by Princeton University Press
41 William Street, Princeton, New Jersey 08540
6 Oxford Street, Woodstock, Oxfordshire OX20 1TR

press.princeton.edu

Library of Congress Control Number: 2019931240
ISBN 978-0-691-16406-9

British Library Cataloging-in-Publication Data is available

Editorial: Vickie Kearn, Susannah Shoemaker, and Lauren Bucca
Production Editorial: Mark Bellis
Text and Jacket Design: Chris Ferrante
Production: Jacqueline Poirier
Publicity: Sara Henning-Stout and Kathryn Stevens
Copyeditor: Alison Durham

Jacket and frontispiece credit: Robert Bosch, Leonardo's *Mona Lisa*,
54 complete sets of double-nine dominos, 2019

This book has been composed in STIX fonts, Gotham, and Alternate Gothic

Printed on acid-free paper. ∞

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

The more constraints one imposes, the more one frees one's self of the chains that shackle the spirit.

IGOR STRAVINSKY

CONTENTS

PREFACE

This book is an account of my ongoing research into how mathematical and computer-science-based optimization techniques can be used to design visual artwork. It contains many equations and inequalities, but all of them are linear, which are the simplest and easiest to understand. It also contains hundreds of images, the vast majority of which I created using the techniques described within.

At the end of the book, there is a long list of references for anyone who wishes to delve deeper into more technical writings on the subject. Many chapters grew out of articles I've written for the Bridges Math/Art conference series and for the Journal of Mathematics and the Arts.

I have benefited enormously from my collaborations with many wonderful, brilliant, and creative people: Kurt Anstreicher, Derek Bosch, Tim Chartier, Martin Chlond, Robert Fathauer, Craig Kaplan, Robert Lang, Doug McKenna, Henry Segerman, Michael Trick, Tom Wexler, and my current and former students Melanie Hart Buehler, Michael Cardiff, Abagael Cheng, Adrienne Herman Cohen, Urchin Colley, Sarah Fries, Gwyneth Hughes, Sage Jenson, Aaron Kreiner, Nikrad Mahdi, Julia Olivieri, Andrew Pike, Mäneka Puligandla, Karen Ressler, Michael Rowan, Harry Rubin-Falcone, Rachael Schwartz, Ari Smith, Jason Smith, Natasha Stout, Elbert Tsai, and Zhifu Xiao.

I am immensely grateful for my colleagues at Oberlin College—its faculty, staff, and students—and my friends in the Bridges and Gathering for Gardner (G4G) communities. I am equally grateful to William Cook's Concorde team and Gurobi Optimization for allowing me to use their extraordinary software packages, and to Vickie Kearn, Susannah Shoemaker, Lauren Bucca, and the rest of the terrific team at Princeton University Press (Mark Bellis, Elizabeth Blazejewski, Alison Durham, Chris Ferrante, Sara Henning-Stout, Dimitri Karetnikov, Meghan Kanabay, Katie Lewis, Jacquie Poirier, Kathryn Stevens, Erin Suydam, and Matthew Taylor) for their expertise, enthusiasm, and patience. Without Oberlin College, Bridges, G4G, Concorde, Gurobi, and PUP, I wouldn't have been able to carry out this project.

I couldn't have even envisioned this project if I hadn't encountered the work of Ken Knowlton, computer graphics pioneer and mosaicist extraordinaire. His domino mosaics blew me away when I first encountered them at age 17, and again at age 21, and then a third time at age 37, when I finally realized that I had acquired the mathematical tools to be able to make them myself.

I wouldn't have had the confidence to commit myself to this project if I hadn't met Annalisa Crannell and Marc Frantz and participated in their 2005 Viewpoints workshop at Franklin and Marshall College.

And I wouldn't have kept at this project without the encouragement of those who commissioned artwork; exhibited my pieces; invited me to give talks or workshops; conversed with me about mathematics, art, or writing; asked me questions that pushed me to dive deeper; evaluated portions of the manuscript; or just told me to keep going. So thank you to the Bosch and Fried families, to Jim and Debbi Walsh, to Laura Albert, Roger Antonsen, Pau Atela, Julie Beier, Nick Bennett, Sharon Blecher, Gail Burton, Case Conover, Bill Cook, Randy Coleman and Rebecca Cross, Simon Ever-Hale, Gwen Fisher, Julian Fleron, Nat Friedman, Joey Gonzalez-Dones, Susan Goldstine, Henry Lionni Guss, James Gyre, George Hart, Allison Henrich, Judy Holdener, Jerry Johnson, Ava Keating, Josh Laison, Cindy Lawrence, Erin McAdams, Doug McKenna, Colm Mulcahy, Mike Naylor, James Peake, Jennifer Quinn, Dana Randall, Reza Sarhangi, J. Cole Smith, David Swart, David Stull, Eve Torrence, Mike Trick, John Watkins, Carolyn Yackel, and the seven anonymous reviewers of my book proposal and manuscript.

This book has been a labor of love, and like most labors of love, it has been fueled by love: the love of my mother, Charlotte Woebcke Bosch (1933–2016); the love of my brother, Derek Bosch; the love of my son, Dima Bosch; and most especially, the love of my best friend, wife, soul mate, and unending source of inspiration, Kathy Bosch.

Opt Art

CHAPTER 1
Optimization and the Visual Arts?

Optimization is the branch of mathematics and computer science concerned with optimal performance, with finding the best way to complete a task. As such, it is extremely applicable, as everyone from time to time attempts to perform some task at the highest level possible. A UPS driver, for instance, may sequence their stops to minimize total distance traveled, time spent on the road, fuel costs, pollutant emissions, or even the number of left turns. Finding an optimal tour, or at least one that is close to optimal, will benefit not only the driver and UPS, but also their customers (through lower prices) and the rest of society (through reduced pollution).

Some optimization problems are easy, while others are extremely difficult. Which is the case depends in large part on the *constraints*—the rules, the restrictions, the limitations—that specify the underlying task. If every stop on the UPS driver's list falls on the same thoroughfare, then finding the optimal route—and proving it to be optimal—is trivial. But if the city is filled with one-way streets, the stops are scattered throughout the city, and some stops must be made during specified time windows, then determining how to perform this task at a high level can require considerable algorithmic ingenuity and computing power.

Optimization has been put to good use in a great number of diverse disciplines: from advertising, agriculture, biology, business, economics, and engineering to manufacturing, medicine, telecommunications, and transportation (to name but a few). Numerous excellent books describe these important, practical applications, and if you turn to the bibliography, you will find my favorites.

The book you hold in your hands is quite different. It is a highly personal account of my more than sixteen-year-long obsession with using mathematical and computer-science-based optimization techniques to create visual artwork. As obsessions go, it is a harmless one, and not nearly as strange as it sounds! Within these pages, I will provide evidence that supports a bold claim: that the mathematical optimizer and the artist have more similarities than differences.

The mathematical optimizer studies problems that involve optimizing—that is, maximizing or minimizing—some quantity of interest (profit or total cost, for example, in business applications). The optimizer's goal is to come up with an optimal solution—perhaps a way of making the profit as large as possible or the total cost as low as possible. In some cases, the optimizer will be satisfied with a *local optimum*, a solution that is better than all neighboring solutions. If you find a local optimum, you can be confident that when you present it to the board, no one sitting there will be able to improve upon your solution by making minor tweaks to it. But in other cases, the optimizer will not rest until they find a *global optimum*, a solution that is provably better than every other solution. If you find a global optimum, you will be able to get a good night's sleep before the board meeting, for you will be certain that no one there—or anywhere—will be able to find a solution that is better than yours.

The artist is also a problem solver and a seeker of high-quality solutions. The creation of a piece of artwork can be considered a problem to be solved. And isn't it difficult to imagine an artist who, when creating a piece, does *not* try to do their best? For some small number of artists, the goal may be to maximize profit, but for most, the goal may be to make the piece as beautiful as possible, or to have as great an emotional impact on viewers as possible. Beauty and emotional impact are impossible to quantify, but haven't we all been in the presence of the critic, the museum-goer, or the gallery-opening shmoozer who in a burst of enthusiasm blurts out something like, "Don't you just *love* this piece? Don't you think that if the artist had added anything more to it, or had left anything out, it would have failed to have the same impact?" (an assertion, to the mathematical optimizer, about local optimality).

Mathematical optimizers are mindful of the roles that constraints play. They know that in some cases, if they impose additional constraints on an optimization problem, the problem will become much more difficult, but in other cases it will become considerably easier. Some constraints seem to be structured in such a way that in their presence, algorithms have trouble working their way to the best part of the *feasible region* (the set of all feasible solutions—the solutions that satisfy all the constraints), whereas other constraints provide the equivalent of handholds and toeholds that form an easily traversed path to optimality.

Artists are similarly mindful. Artists are well aware that they must deal with constraints. They must work within budgets. They must meet deadlines. If they enter competitions or juried shows, they must make sure that their pieces satisfy the rules of entry. If they take commissions, they must follow their clients' instructions. And no matter what media they choose to work with, they must deal with the particular constraints—imposed by the laws of physics—that govern how those media work. Painting with watercolors is different from painting with oils, and painting on rice paper is different from painting on canvas.

So, given that artists are creative, we might think that if it were up to them, they would do away with constraints. After all, constraints *constrain*. They restrict. They limit our choices. It would seem that constraints inhibit creativity.

But actually there is much evidence to the contrary. Many artists embrace constraints. Some need deadlines to be able to finish their work, and some believe that when their choices are limited, they are much more focused and creative. Joseph Heller (while paraphrasing T. S. Eliot) wrote,

> When forced to work within a strict framework the imagination is taxed to its utmost—and will produce its richest ideas.

And the psychologist Rollo May wrote,

> Creativity arises out of the tension between spontaneity and limitations, the latter (like the river banks) forcing the spontaneity into the various forms which are essential to the work of art or poem.

In fact, many artists go so far as to create their own constraints. Consider George-Pierre Seurat. While viewing his painting *A Sunday on La Grande Jatte–1884* from up close, one sees a mass of colorful dots. While backing away from it, one's eyes merge all of the dots into an image of a group of Parisians relaxing on an island on the Seine. To create this masterpiece, Seurat set himself the task of producing the best possible depiction of what he saw on the riverbank, subject to two highly restrictive, self-imposed constraints: he had to keep his colors separate, and he could only apply paint to the canvas with tiny, precise, dot-like brush strokes. Seurat's self-imposed constraints gave rise to a spectacular piece of artwork, the most widely reproduced example of what we now call Pointillism.

In the mosaicking arena, self-imposed constraints abound. Every time a mosaicist states, "I will build a mosaic out of ______," another self-imposed constraint is born (or at least conceived). In 400 BCE, the ancient Greeks were building mosaics out of differently colored pebbles, and around 200 BCE, they started building them out of specially manufactured tiles (tesserae) made out of ceramic, stone, or glass. Today's mosaicists still use these traditional materials, but they also use whatever else they have on hand: dice, dominos, LEGO bricks, Rubik's Cubes, toy cars, spools of thread, baseball cards, photographs, and even individual frames of films like *Star Wars* and *It's a Wonderful Life*.

Some mosaicists like to go beyond the inherent materials constraints. The domino mosaics of Ken Knowlton, Donald Knuth, and myself are not only made out of dominos, they are made out of *complete sets* of dominos. Knowlton's *Joseph Scala (Domino Player)* (from 1981) was made out of 24 complete sets of double-nine dominos, so it contains 24 dominos of each type: exactly 24 blank dominos, exactly 24 zero-one dominos, and so on. My domino portrait of President Obama, the 44th president of the United States, uses 44 complete sets. Knowlton's portrait of Helen Keller is composed of the 64 characters of the Braille writing system, and each of these characters appears 16 times. Chris Jordan's *Denali/Denial* mosaic arranges 24,000 (digitally altered) logos from the GMC Yukon Denali sports utility vehicle (six weeks of sales in 2004) into an image of Denali (also known as Mount McKinley). And a Robert Silvers photomosaic, commissioned by *Newsweek* for its 1997 pictures-of-the-year issue, portrays the late Princess Diana as a mosaic formed from thousands of photographs of flowers. All of these artists use computer software—usually computer programs that they have developed themselves—to design their mosaics.

In theory, you can design a photomosaic without software. You can take the senior portrait photos from your high-school yearbook, cut them out, and then arrange them in a rectangular grid so that from a distance, they will collectively resemble a photo of your favorite teacher. This is possible—some of the photos will be brighter and others will be darker. But you will need a good eye to assess the brightness of each photo, and even then, you will have a tough time determining the best position for each photo. Likewise, you can make a domino mosaic without software—by printing the target image on a large piece of paper and then placing dominos on top of the print, saving the brightest dominos (the nine-nines) for the brightest sections and the darkest dominos (the zero-zeros or blanks) for the darkest sections. Here, though it is clear which dominos are brighter than others, it still will be difficult to determine where to place each domino.

With mathematical optimization it is quite easy to design photomosaics, and it isn't all that difficult to design domino mosaics. With mathematical optimization, the artist/mathematician (or mathematician/artist) can explore all manner of constraints systems. This book is an account of my explorations of this world.

CHAPTER 2

Truchet Tiles

Father Sébastien Truchet (1657–1729) entered the Carmelite order at age 17 and impressed his superiors with a genius for all things mechanical. Sent to Paris to further his education, the brilliant Truchet drew the attention of Louis XIV's men after Charles II of England had given the French king two watches and neither the Sun King nor his royal watchmaker could open them. The then-19-year-old Truchet was consulted, and he quickly discovered how to work the mechanism and repair the damage caused by the previous attempts at unlocking it.

For this success, Louis XIV awarded Truchet a sizable pension, which enabled him to throw himself completely into the study of "first the geometry necessary for the theory of mechanics and even anatomy and chemistry, neglecting nothing of what might be useful with respect to machines." And by the time of his death, Truchet was known both as France's foremost expert in hydraulics engineering and as a prodigious inventor. At the request of Louis XIV, he had worked on the aqueduct system of Versailles and had been involved in the construction of, or repair of, most of the French canals. One of his inventions was a machine that could transport whole trees without damaging them. Another, a team effort, was the infinitely scalable Romain du Roi typeface.

But today Truchet is not remembered for these great accomplishments. Instead he is known for a set of tiles, displayed in figure 2.1,

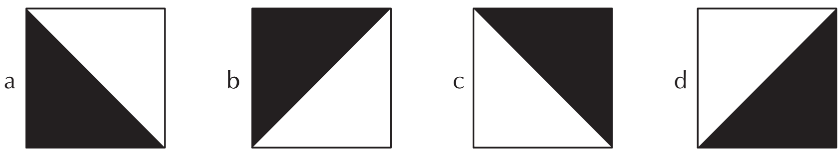

Figure 2.1: Truchet tiles.

that caught his eye while he was in the city of Orleans inspecting canals.

At first glance it is surprising that these simple square tiles, each divided by a diagonal into a white half and a black half, not only captured the attention of Truchet's brilliant mind, but then held it long enough to inspire him to write an article, "Mémoire sur les combinaisons," and submit it for publication in the most prestigious academic journal of his time, *Memoires de l'Académie Royale de Sciences*, in 1704. But Truchet's article makes it clear that his fascination was less with the tiles themselves than with how they can be combined to form larger patterns. He devoted the bulk of the article to beautifully engraved plates that display "the fecundity of these combinations, the origin of which is nevertheless so very simple."

Figure 2.2 reproduces four of Truchet's patterns, along with the labels he gave them. Pattern A uses only tiles of type a, so we can say that pattern A is generated by tile a, and we can express this by writing $A=(a)$. Pattern C is a checkerboard formed from tiles of types a and c. Its odd numbered rows begin

$$a \quad c \quad a \quad c \quad \ldots,$$

and its even numbered rows begin

$$c \quad a \quad c \quad a \quad \ldots.$$

If we focus our attention on 2-by-2 blocks of tiles, starting at the top-left corner, we see that pattern C is generated by the 2-by-2 block $\left(\begin{smallmatrix} a & c \\ c & a \end{smallmatrix}\right)$. We can express this by writing $C=\left(\begin{smallmatrix} a & c \\ c & a \end{smallmatrix}\right)$. Like C, pattern D has a 2-by-2 generator, but unlike C it uses all four types of tiles. We can write pattern D as $D=\left(\begin{smallmatrix} b & a \\ c & d \end{smallmatrix}\right)$. Pattern E has a 4-by-4 generator.

Truchet's article displays 26 additional patterns, ordered by increasing complexity, and a 1722 book written by his Carmelite colleague Father Dominique Doüat shows many more. Doüat's pattern 72, reproduced in figure 2.3, is much busier than Truchet's patterns A, C, D, and E. Part of the reason is that it has a much larger generator, a 12-by-12 block of tiles.

Doüat was absolutely enthralled by Truchet's tiles. He titled his book *Methode pour fair une infinité de desseins différens, avec des carreaux mi-partis de deux couleurs par une ligne diagonal* (Method for making