

Fat-Tailed and Skewed Asset Return Distributions

*Implications for Risk Management,
Portfolio Selection, and Option Pricing*

SVETLOZAR T. RACHEV

CHRISTIAN MENN

FRANK J. FABOZZI



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Preface

The theory and practice of finance draws heavily on probability theory. All MBA programs prepare finance majors for their career in the profession by requiring one generalist course in probability theory and statistics attended by all business majors. While several probability distributions are covered in the course, the primary focus is on the normal or Gaussian distribution.

Students find it easy to understand and apply the normal distribution: Give them the expected value and standard deviation and probability statements about outcomes can be easily made. Moreover, even if a random variable of interest is not normally distributed, students are told that a theorem in statistics called the *Central Limit Theorem* proves that under certain conditions the sum of independent random variables will be asymptotically normally distributed. Loosely speaking, this means that as the number of random variables are summed, the sum will approach a normal distribution.

Armed with this rudimentary knowledge of probability theory, finance students march into their elective courses in finance that introduce them to the quantitative measures of risk (the standard deviation) and the quantitative inputs needed to implement modern portfolio theory (the expected value or mean and the standard deviation). In listing assumptions for most theories of finance, the first assumption on the list is often: “Assume asset returns are normally distributed.” The problem, however, is that empirical evidence does not support the assumption that many important variables in finance follow a normal distribution. The application of the Central Limit Theorem to such instances is often inappropriate because the conditions necessary for its application are not satisfied.

And this brings us to the purpose of this book. Our purpose is four-fold. First, we explain alternative probability distributions to the normal distributions for describing asset returns as well as defaults. We focus on the stable Paretian (or alpha stable) distribution because of the strong support for that distribution that dates back four decades to the seminal work of Benoit Mandelbrot. Second, we explain how to estimate distributions. Third, we present empirical evidence rejecting the hypothesis that returns for stocks and bonds are normally distributed

and instead show that they exhibit fat tails and skewness. Finally, we explain the implications of fat tails and skewness to portfolio selection, risk management, and option pricing.

We must admit that our intent at the outset was to provide a “non-technical” treatment of the topic. However, we could not do so. Rather, we believe that we have provided a less technical treatment than is provided in the many excellent books and scholarly articles that deal with probability and statistics applied to finance and risk management. The book is not simple reading. It must be studied to appreciate the pitfalls that result from the application of the normal distribution to real-world financial problems.

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Svetlozar T. Rachev
Christian Menn
Frank J. Fabozzi

About the Authors

Professor Dr. Svetlozar T. (Zari) Rachev is Chair-Professor at the University of Karlsruhe in the School of Economics and Business Engineering and Professor Emeritus at the University of California, Santa Barbara in the Department of Statistics and Applied Probability. He completed his PhD. in 1979 from Moscow State University and his Doctor of Science Degree in 1986 from the Steklov Mathematical Institute in Moscow. He has published six monographs and more than 250 research articles. His research areas include mathematical and empirical finance, econometrics, probability, and statistics. He is a Fellow of the Institute of Mathematical Statistics, Elected Member of the International Statistical Institute, Foreign Member of the Russian Academy of Natural Science, and holds an honorary doctorate degree from St. Petersburg Technical University. Professor Rachev is cofounder of Bravo Risk Management Group, specializing in financial risk-management software. Bravo Group was recently acquired by FinAnalytica for which he currently serves as Chief-Scientist.

Dr. Christian Menn is Hochschulassistent at the Chair of Statistics, Econometrics and Mathematical Finance at the University of Karlsruhe. Currently, he is Visiting Scientist at the School of Operations Research and Industrial Engineering at Cornell University as a post-doctorate fellow. In 1998, he received a degree in mathematics (Maîtrise des mathématiques) from the University J. Fourier in Grenoble, France and in 2000 a degree in mathematics (Diplom in Wirtschaftsmathematik) from the University of Karlsruhe, Germany. Recently, Christian earned his doctorate in economics at the University of Karlsruhe, Germany.

Dr. Frank J. Fabozzi, CFA, CPA is the Frederick Frank Adjunct Professor of Finance in the School of Management at Yale University. Prior to joining the Yale faculty, he was a Visiting Professor of Finance in the Sloan School at MIT. Professor Fabozzi is a Fellow of the International Center for Finance at Yale University and the editor of the *Journal of Portfolio Management*. He earned a doctorate in economics from the City University of New York in 1972. In 1994 he received an honorary doctorate of Humane Letters from Nova Southeastern University and in 2002 was inducted into the Fixed Income Analysts Society's Hall of Fame.