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*Dedicated to  
our parents and family members*

## Preface

Lot sizing is one of the most important decisions taken during production planning in all manufacturing and process industries. Among the various basic lot sizing models, the Capacitated Lot Sizing Problem (CLSP) is the main focus of this book. A single-machine, single-level and multiple-item CLSP is considered. It is the problem of planning the production of several items over a number of periods, satisfying all demand requirements so that the production times and setup times do not exceed the capacity limitations, and the total cost of the production plan is minimized. The main cost components are set up cost, inventory holding cost, backorder cost and lost-sales cost. The CLSP comes under the class of big-bucket lot sizing problems in the CLSP literature. It is considered as a big-bucket problem because several products/setups may be produced in a period. A period normally represents a time slot of one shift/day or one day or one week or one month (depending upon the production planner). For each lot produced in a period, a cost of setup occurs, and the time corresponding to the setup along with the production time (product of the production quantity and the number of units of a product produced per unit time considered) consumes the capacity in a period (assumed in time units). The CLSP is solved over a finite time horizon due to which excess quantity produced in a period can be stored to satisfy the demand of some future period, and the demand which cannot be met in a period due to capacity constraints can be backordered. Following the basic CLSP, studies on lot sizing consider the phenomenon where the setup state of a product is carried from one period to another in order to avoid multiple setups for the same product in consecutive periods. This phenomenon is called as setup carryover in the literature, whereas it is called production carryover in this book.

In almost all process industries, there are situations where some products have long and uninterrupted setup times, and the setup of the product and its consecutive production can be carried over across consecutive periods. Also, certain process industries require the production of a product to occur immediately after its setup, and the product to be continuously produced without any interruption. The phenomenon where the setup of a product having long setup time is carried over across periods is called setup crossover in the literature as well as in this book.

In this book, a mathematical model for the Capacitated Lot Sizing Problem with Production Carryover and Setup Crossover across periods (CLSP-PCSC), with possible backorders and with such real-life considerations in process industries, is proposed in Chap. 3. The aspect of allowing the setup to be carried over more than one period is called setup crossover in this book. The model proposed is all encompassing that it can handle continuous manufacturing (as in the case of process industries), and also situations where the setup costs and holding costs are product dependent and time independent/time dependent, with appropriate adaptations. The proposed model is also compared with an existing MILP (Mixed Integer Linear

Programming) model. A heuristic is also proposed to solve the CLSP-PCSC. The performance of the mathematical model and the heuristic is presented.

In Chap. 4, another mathematical model and a comprehensive heuristic are proposed for the same problem. This model is also all encompassing in that it can handle continuous manufacturing (as in process industries), and also situations where the setup costs and holding costs are product dependent and time independent/time dependent, with appropriate adaptations. A comprehensive heuristic is proposed based on this mathematical model to solve the CLSP-PCSC. The performance of the proposed model and the heuristic is evaluated using problem instances of various sizes.

In some process and manufacturing industries, the setup time of a machine not only depends on the time to setup a product but it also depends upon the product previously setup on the machine. This aspect where the setup time of a product on the machine depends on the time to setup a given product after setting up a given preceding product is called the sequence-dependent setup. The corresponding time taken for setting up the product is called as the sequence-dependent setup time and the corresponding cost involved is called the sequence-dependent setup cost. Researchers have considered the presence of sequence-dependent setup times and setup costs while addressing CLSP in industries in the presence of sequence-dependent setups. This book presents in Chap. 5 mathematical models developed for the Capacitated Lot Sizing Problem with Production Carryover and Setup Crossover across periods, assuming Sequence-Dependent Setup Times and Setup Costs (CLSP-SD-PCSC). In addition, these models allow the presence of backorders and also address real-life situations present in process industries such as the production of a product starting immediately after its uninterrupted setup and the uninterrupted production carryover across periods, along with the presence of long setup times. The consideration of these real-life situations is unique to this book and is one of its most significant contributions in every chapter.

In summary, this book addresses a class of CLSPs addressing some real-life situations present in process industries. Several variants of mathematical models and heuristics are proposed and developed to address some classes of lot sizing problems.

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|      |                                                                                                                                                                                                                                                                                                                                                                                                                            |     |
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## ABBREVIATIONS

|                         |                                                                                                                                                |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>APS</b>              | Advanced Planning Systems                                                                                                                      |
| <b>B&amp;B</b>          | Branch & Bound                                                                                                                                 |
| <b>BoM</b>              | Bill of Materials                                                                                                                              |
| <b>CCO</b>              | Compressing Carry Over model                                                                                                                   |
| <b>CLSD</b>             | Capacitated Lot Sizing Problem with Sequence-Dependent Setups                                                                                  |
| <b>CLSP</b>             | Capacitated Lot Sizing Problem                                                                                                                 |
| <b>CLSPL</b>            | Capacitated Lot Sizing Problem with Linked lot sizes                                                                                           |
| <b>CLSP-PCSC</b>        | Capacitated Lot Sizing Problem with Production Carryover and Setup Crossover                                                                   |
| <b>CLSP-SD-PCSC</b>     | Capacitated Lot Sizing Problem with Sequence-Dependent Setups along with Production Carryover and Setup Crossover                              |
| <b>CNC</b>              | Computer Numeric Control                                                                                                                       |
| <b>CO</b>               | Carry Over model                                                                                                                               |
| <b>CSLP</b>             | Continuous Setup Lot sizing Problem                                                                                                            |
| <b>DLSP</b>             | Discrete Lot sizing and Scheduling Problem                                                                                                     |
| <b>ELSP</b>             | Economic Lot Scheduling Problem                                                                                                                |
| <b>EOQ</b>              | Economic Order Quantity                                                                                                                        |
| <b>EDI</b>              | Electronic Data Interchange                                                                                                                    |
| <b>ERP</b>              | Enterprise Resource Planning                                                                                                                   |
| <b>FL</b>               | Facility Location                                                                                                                              |
| <b>FMS</b>              | Flexible Manufacturing System                                                                                                                  |
| <b>GA</b>               | Genetic Algorithm                                                                                                                              |
| <b>GRASP</b>            | Greedy Randomized Adaptive Search Procedure                                                                                                    |
| <b>LP</b>               | Linear Programming                                                                                                                             |
| <b>LUC</b>              | Least Unit Cost                                                                                                                                |
| <b>MES</b>              | Manufacturing Execution Systems                                                                                                                |
| <b>MILP</b>             | Mixed Integer Linear Programming                                                                                                               |
| <b>MIP</b>              | Mixed Integer Programming                                                                                                                      |
| <b>MLCLSP</b>           | Multi-Level Capacitated Lot Sizing Problem                                                                                                     |
| <b>MM1:CLSP-PCSC</b>    | Mathematical Model 1 for the Capacitated Lot Sizing Problem with Production Carryover and Setup Crossover                                      |
| <b>MM2:CLSP-PCSC</b>    | Mathematical Model 2 for the Capacitated Lot Sizing Problem with Production Carryover and Setup Crossover                                      |
| <b>MM1:CLSP-SD-PCSC</b> | Mathematical Model 1 for the Capacitated Lot Sizing Problem with Sequence-Dependent Setups along with Production Carryover and Setup Crossover |

|                         |                                                                                                                                                |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>MM2:CLSP-SD-PCSC</b> | Mathematical Model 2 for the Capacitated Lot Sizing Problem with Sequence-Dependent Setups along with Production Carryover and Setup Crossover |
| <b>MRP</b>              | Material Requirements Planning                                                                                                                 |
| <b>NCO</b>              | Non-Carry Over model                                                                                                                           |
| <b>NP</b>               | Non-Polynomial                                                                                                                                 |
| <b>PLSP</b>             | Proportional Lot sizing and Scheduling Problem                                                                                                 |
| <b>POQ</b>              | Periodic Order Quantity                                                                                                                        |
| <b>PPB</b>              | Part Period Balancing                                                                                                                          |
| <b>SA</b>               | Simulated Annealing algorithm                                                                                                                  |
| <b>SMC</b>              | Simple Multi-Commodity                                                                                                                         |
| <b>SR</b>               | Shortest Route                                                                                                                                 |
| <b>SSR</b>              | Strengthened Shortest Route                                                                                                                    |

## Notations

|                 |                                                                                                                                                                                      |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $N$             | Number of products                                                                                                                                                                   |
| $T$             | Number of time periods                                                                                                                                                               |
| $t$             | Time period                                                                                                                                                                          |
| $i$             | Product                                                                                                                                                                              |
| $SC_i$          | Setup cost for product $i$                                                                                                                                                           |
| $SC_{i,t}$      | Setup cost for product $i$ , when its setup is initiated in period $t$ ; this cost is incurred only once as a fixed cost computed with respect to the period of its setup initiation |
| $SC_{i,t}^1$    | Rate of cost of setup (cost corresponding to one time unit of setup) of product $i$ in period $t$ . It is given by $SC_{i,t}^1 = \frac{SC_{i,t}}{ST_i}$                              |
| $SC_{\phi,i}$   | Sequence-dependent setup cost incurred in the machine for the first product $i$ setup in period 1                                                                                    |
| $SC_{i',i}$     | Sequence-dependent setup cost incurred when the machine is set up from product $i'$ to product $i$                                                                                   |
| $h_i$           | Holding cost per period per unit of product $i$                                                                                                                                      |
| $h_{i,t}$       | Holding cost per period per unit of product $i$ in period $t$                                                                                                                        |
| $b_i$           | Backorder cost per period per unit of product $i$                                                                                                                                    |
| $ST_i$          | Setup time for product $i$                                                                                                                                                           |
| $ST_{\phi,i}$   | Sequence-dependent setup time for the first product $i$ setup in the machine in period 1                                                                                             |
| $ST_{i',i}$     | Sequence-dependent setup time when the machine is set up from product $i'$ to product $i$                                                                                            |
| $ST_{i,t}'$     | A variable that is assigned with the value of setup time of product $i$ which is set up in period $t$                                                                                |
| $a_i$           | Number of time units required for producing one unit of product $i$                                                                                                                  |
| $C_t$           | Capacity of the machine in period $t$ (in time units)                                                                                                                                |
| $d_{i,t}$       | Demand for product $i$ in period $t$                                                                                                                                                 |
| $M$             | A large value                                                                                                                                                                        |
| $\mathcal{E}$   | Smallest unit of time (a small positive real number)                                                                                                                                 |
| $\mathcal{E}_d$ | Unit of smallest quantity of production (a small positive real number)                                                                                                               |

|           |                                                            |
|-----------|------------------------------------------------------------|
| $I_{i,t}$ | Inventory of product $i$ at the end of period $t$          |
| $B_{i,t}$ | Backorder quantity of product $i$ at the end of period $t$ |

**Definition of Variables Specific to Mathematical Model (MM1:CLSP-PCSC) in Chap. 3**

| Variable            | Description                                                                                                                                                                                                                                                                              |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\delta_{i,t}^1$    | An indicator (binary) variable that takes value 1 if a complete setup is done for product $i$ in period $t$ with the production starting in period $t$ ; 0 otherwise                                                                                                                     |
| $\Delta_{i,t,t'}^1$ | An indicator (binary) variable that takes value 1: it corresponds to the production carryover from period $t$ to period $t'$ ( $t \leq t' \leq T$ ), due to the setup of product $i$ started and finished in period $t$ , with no intermittent setup of any other product; 0 otherwise   |
| $\delta_{i,t}^2$    | An indicator (binary) variable that takes value 1 if a setup of product $i$ is started and completed exactly at the end of period $t$ , followed by its production starting in period $t + 1$ ; 0 otherwise                                                                              |
| $\Delta_{i,t,t'}^2$ | An indicator (binary) variable that takes value 1: it corresponds to the production carryover from period $t'$ to period $t' + 1$ ( $t + 1 \leq t' \leq T$ ), due to the end-of-period setup of product $i$ in period $t$ , with no intermittent setup of any other product; 0 otherwise |
| $\delta_{i,t,t'}^3$ | An indicator (binary) variable that takes value 1 if the setup of product $i$ is commenced in period $t$ and is completed during some period $t'$ but not exactly at the end of period $t'$ ( $t + 1 \leq t' \leq T$ ); 0 otherwise                                                      |

| Variable                | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\Delta_{i,t,t',t''}^3$ | An indicator (binary) variable that takes value 1: it corresponds to the production in period $t''$ ( $t' \leq t'' \leq T$ ), due to the setup of product $i$ initiated in period $t$ and completed in period $t'$ but not exactly at the end of period $t'$ ( $t + 1 \leq t' \leq T$ ) and with no setup of any product during the intermittent periods from period $t'$ to period $t''$ . <i>Note:</i> This variable corresponds to (i.e. indicates) the production carryover through periods $t'$ and $t''$ , after the completion of setup in period $t'$ (but not exactly at the end of period $t'$ ), with the initiation of setup in period $t$ ;<br>0 otherwise |
| $\delta_{i,t,t'}^4$     | An indicator (binary) variable that takes value 1 if the setup of product $i$ is commenced in period $t$ and is completed exactly at the end of period $t'$ ( $t+1 \leq t' \leq T-1$ );<br>0 otherwise                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| $\Delta_{i,t,t',t''}^4$ | An indicator (binary) variable that takes value 1: it corresponds to the production in period $t''$ ( $t' + 1 \leq t'' \leq T$ ), due to the setup of product $i$ initiated in period $t$ and completed exactly at the end of period $t'$ ( $t + 1 \leq t' \leq T - 1$ ) and with no setup of any product during the intermittent periods from period $t'$ to period $t''$ . <i>Note:</i> This variable corresponds to (i.e. indicates) the production carryover through periods $t'$ and $t''$ , after the completion of setup exactly at the end of period $t'$ , with the initiation of setup in period $t$ ;<br>0 otherwise                                         |
| $s_{i,t}^1$             | Setup time of product $i$ in period $t$ that takes the value of $ST_i$ , and associated with $\delta_{i,t}^1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| $s_{i,t}^2$             | Setup time of product $i$ in period $t$ that takes the value of $ST_i$ , and associated with $\delta_{i,t}^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| $s_{i,t,t',t''}^3$      | Setup time of product $i$ in period $t''$ ( $t \leq t'' \leq t'$ ), when its setup has started in period $t$ and completed in period $t'$ , but not completed exactly at the end of period $t'$ , and associated with $\delta_{i,t,t'}^3$ ; <i>Note:</i> $\sum_{t''=t}^{t'} s_{i,t,t',t''}^3 = ST_i$                                                                                                                                                                                                                                                                                                                                                                    |
| $s_{i,t,t',t''}^4$      | Setup time of product $i$ in period $t''$ ( $t \leq t'' \leq t'$ ), when its setup has started in period $t$ and completed exactly at the end of period $t'$ , and associated with $\delta_{i,t,t'}^4$ ; <i>Note:</i> $\sum_{t''=t}^{t'} s_{i,t,t',t''}^4 = ST_i$                                                                                                                                                                                                                                                                                                                                                                                                       |

| Variable           | Description                                                                                                                                                                                                                                                                                                    |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $X_{i,t,t'}^1$     | Production quantity of product $i$ in period $t'$ (due to its setup started and ended in period $t$ ), with $1 \leq t \leq T$ and $t \leq t' \leq T$ , and associated with $\Delta_{i,t,t'}^1$                                                                                                                 |
| $X_{i,t,t'}^2$     | Production quantity of product $i$ in period $t'$ (due to its setup started in period $t$ and completed exactly at the end of period $t$ ), with $1 \leq t \leq T-1$ and $t+1 \leq t' \leq T$ , and associated with $\Delta_{i,t,t'}^2$                                                                        |
| $X_{i,t,t',t''}^3$ | Production quantity of product $i$ in period $t''$ (due to its setup started in period $t$ and ended in period $t'$ but not completed at the end of period $t'$ ), with $1 \leq t \leq T-1$ , $t+1 \leq t' \leq T$ and $t+1 \leq t'' \leq T$ , and $t'' \geq t'$ , and associated with $\Delta_{i,t,t',t''}^3$ |
| $X_{i,t,t',t''}^4$ | Production quantity of product $i$ in period $t''$ (due to its setup started in period $t$ and completed at the end of period $t'$ ), with $1 \leq t \leq T-2$ , $t+1 \leq t' \leq T-1$ and $t+2 \leq t'' \leq T$ , and $t'' > t'$ , and associated with $\Delta_{i,t,t',t''}^4$                               |

**Definition of Variables Specific to Mathematical Model (MM2:CLSP-PCSC) in Chap. 4**

| Variable            | Description                                                                                                                                                                                                                                                                                      |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\delta_{i,t}^1$    | An indicator (binary) variable that takes value 1 if a complete setup is done for product $i$ in period $t$ with the production starting in period $t$ ;<br>0 otherwise                                                                                                                          |
| $\Delta_{i,t,t'}^1$ | An indicator (binary) variable that takes value 1: it corresponds to a possible production carryover from period $t$ to period $t'$ ( $t \leq t' \leq T$ ), due to the setup of product $i$ started and finished in period $t$ , with no intermittent setup of any other product;<br>0 otherwise |
| $\delta_{i,t}^2$    | An indicator (binary) variable that takes value 1 if the setup of product $i$ is started and completed exactly at the end of period $t$ followed by its production starting in period $t+1$ ;<br>0 otherwise                                                                                     |
| $\Delta_{i,t,t'}^2$ | An indicator (binary) variable that takes value 1: it corresponds to a possible production carryover from period $t'$ to period $t'+1$ ( $t+1 \leq t' \leq T$ ), due to the end-of-period setup of product $i$ in period $t$ , with no intermittent setup of any other product;<br>0 otherwise   |

| Variable            | Description                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\delta_{i,t}^3$    | An indicator (binary) variable that takes value 1 if the setup of product $i$ is commenced in period $t$ and is carried over across periods, and is completed in some period $t'$ ( $t' = t + 1, t + 2, \dots, T$ );<br>0 otherwise                                                                                                                                                                                                            |
| $\Omega_{i,t,t'}^3$ | An indicator (binary) variable that takes value 1 if the setup of product $i$ is initiated in period $t$ and is present in period $t'$ ( $t \leq t' \leq T$ ), with the setup of product $i$ ending in a period later than period $t$ , but not exactly at the end of that period;<br>0 otherwise                                                                                                                                              |
| $\Omega_{i,t,t'}^4$ | An indicator (binary) variable that takes value 1 if the setup of product $i$ is initiated in period $t$ and is present in period $t'$ ( $t \leq t' \leq T - 1$ ), with the setup of product $i$ ending in a period later than period $t$ and setup getting completed exactly at the end of that period;<br>0 otherwise                                                                                                                        |
| $\Delta_{i,t,t'}^3$ | An indicator (binary) variable that takes value 1: it corresponds to a possible production carryover from period $t'$ to period $t' + 1$ ( $t + 1 \leq t' \leq T$ ), with the setup of product $i$ (having started in period $t$ ) ending in a period later than period $t$ , but not exactly at the end of that period, and with no intermittent setup of any product during the production of product $i$ ;<br>0 otherwise                   |
| $\Delta_{i,t,t'}^4$ | An indicator (binary) variable that takes value 1: it corresponds to a possible production carryover from period $t'$ to period $t' + 1$ ( $t + 2 \leq t' \leq T$ ), with the setup of product $i$ (having started in period $t$ ) ending in a period later than period $t$ and setup getting completed exactly at the end of that period, and with no intermittent setup of any product during the production of product $i$ ;<br>0 otherwise |
| $s_{i,t}^1$         | Setup time of product $i$ in period $t$ that takes the value of $ST_i$ , and associated with $\delta_{i,t}^1$                                                                                                                                                                                                                                                                                                                                  |
| $s_{i,t}^2$         | Setup time of product $i$ in period $t$ that takes the value of $ST_i$ , and associated with $\delta_{i,t}^2$                                                                                                                                                                                                                                                                                                                                  |
| $s_{i,t,t'}^3$      | Setup time of product $i$ in period $t'$ due to its setup started in period $t$ , and associated with $\Omega_{i,t,t'}^3$                                                                                                                                                                                                                                                                                                                      |