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Modelling and Control Design of Vibration Reduction Systems

Methods and Procedures of Selecting Vibro-isolation Properties

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Preface

Scope

The aim of the book is to develop and verify appropriate methods and procedures for selecting the vibro-isolation properties of vibration reduction systems which is enabled to evaluate the visco-elastic characteristics of passive systems and support the control system design of semi-active and active systems. The generalised methods and procedures should allow to select the system vibro-isolation properties for different classes of the oscillatory systems in terms of the chosen optimisation criteria (usually conflicted criteria). The elaborated methods and procedures enable to protect the working machine operators from the adverse effects of vibration exposure.

Content

At first, spectral classes of the excitation signals are determined that are representative for different vibrating systems. This is a basis of shaping the vibro-isolation properties of vibration reduction systems. Then, the generalised mathematical models of vibration isolation systems are developed. These models enable to shape non-linear characteristics of the visco-elastic elements included in vibration isolator. Such a mathematical description of the vibration reduction systems determines the ability to use the developed models in a wide range of applications. Successively, the reliable and efficient optimisation criteria are determined and these criteria are frequency weighted in order to indicate the resonances of human body parts and organs.

Secondly, a method of shaping the vibro-isolation properties of passive vibration reduction systems is elaborated and such a method allows to determine non-linear characteristics of the basic elements applied in the system. This is achieved using the optimisation procedures that are able to find compromise solutions

(Pareto-optimal) with respect to conflicted vibro-isolation criteria. The effectiveness of proposed method is verified experimentally using the exemplary, passive suspension system that is excited using the signals of various spectral classes.

Finally, a method of shaping the vibro-isolation properties of semi-active and active systems is elaborated and such a method facilitates the control design of technically advanced vibration reduction systems. Using the proposed optimisation procedures, it is possible to adjust the vibro-isolation properties by changing the controller settings. The designed control system using the proposed method is investigated experimentally by means of the exemplary, semi-active and active suspension systems that are loaded of different masses.

Acknowledgements

The following work is a part of the research project 'Methods and procedures of selecting vibro-isolation properties of vibration reduction systems' founded by the National Science Center of Poland under the contract No. UMO-2013/11/B/ST8/03881. We would like to thank ISRINGHAUSEN GmbH & Co. KG for the assistance in the experimental research.

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Chapter 1
Introduction

1.1 State of the Art

There are two basic sources of mechanical vibrations that can disturb a proper functioning of the machines. The first one relates to the systems that during their operation generate vibrations (Fig. 1.1a), e.g. engine vibrations in the working machine. The second class concerns the systems that vibrate due to external reactions (Fig. 1.1b), e.g. cab vibrations of the machine which is moving over uneven ground [1, 2]. Many sources of the vibrations can cause periodic or random reactions of the machine elements. In such situation, resonant states can be obtained and they may cause movement disturbances of individual machine elements. The resonant vibrations have an adverse effect on machine functioning, and this can lead to their failure. In addition, the vibrations have a negative influence on the health condition of working machine operators [3].

In most cases, the vibrations are harmful processes, having a detrimental impact on the human operator of a machine. The danger arising from the exposure of a human body to vibrations increases in the case of high-intensity vibrations with a prolonged duration period. Figure 1.2 gives the root mean square (RMS) acceleration values of the vibrations to which the operators of the most popular heavy machinery are exposed during their work [4]. The data presented are only to visualise the ranges of accelerations of the vibrations that induce operators' movements.

Earth-moving machines and most other forms of engineering vehicle expose their operators to whole-body vibration. According to the ISO-2631 standard [5], the criterion employed to evaluate the intensity of vibration transmitted to a human is based on the length of time (in hours) over which an exposure to vibration results in no health risks (NH), potential health risks (PR) or likely health risks (HR). For better presentation, the curves defining these three levels of the vibration exposure are shown in Fig. 1.3.

There are several methods to minimise vibrations which have a negative influence on working machines operators during their work. The first group of methods

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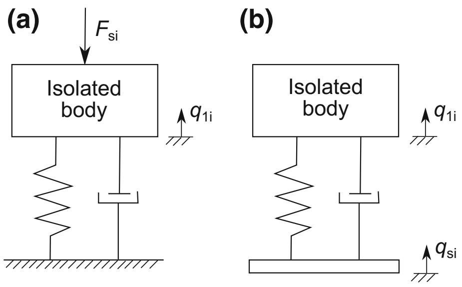

Fig. 1.1 Operating equipment generating a disturbance force F_{si} (**a**) and sensitive equipment supported by a vibrating structure q_{si} (**b**)

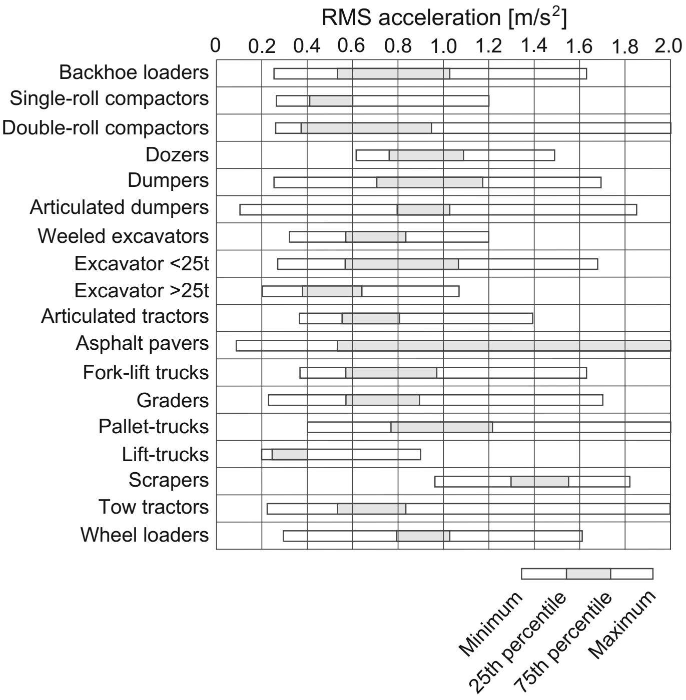

Fig. 1.2 Root mean square (RMS) acceleration of vibration obtained from more than 20 measurements carried out for each of the machine types [4]

comprises the passive techniques, and these techniques can be introduced in the following way [6]:

- prevention of vibration,
- structural modifications,
- parametric modifications,
- damping of vibration.

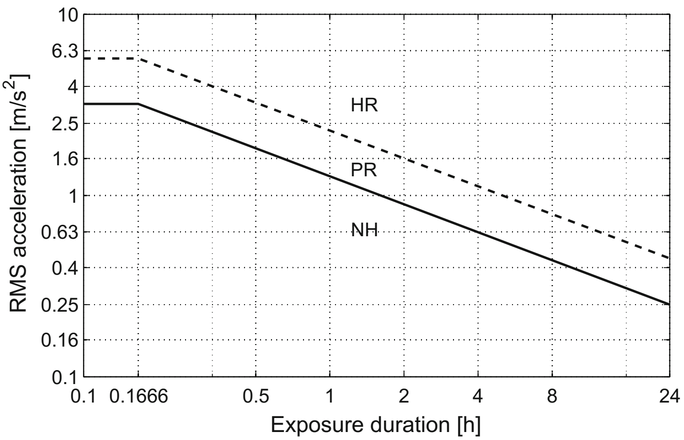

Fig. 1.3 Maximum vibration exposure time (hours) that corresponds to no health risk condition (NH), potential health risk (PR) and health risk is likely (HR) in accordance with the ISO-2631 standard [5]

In the recent literature, [3, 7, 8] only simplified, linear models of the vibration reduction systems are discussed. However, it should be noted that many vibration isolators demonstrate non-linear stiffness (e.g. pneumatic spring) and damping characteristics (e.g. hydraulic shock-absorber). The stiffness of suspension system is non-linear in the relative displacement domain, and its characteristic has a dissimilar shape for different loading masses (Fig. 1.4a). The damping of suspension system is also non-linear in the relative velocity domain, and its characteristic can be changed by the machine operator using the control lever (Fig. 1.4b). These considerations lead to the conclusion that linear mathematical description of many modelled vibration isolators is possible only with some approximation around a system operating point.

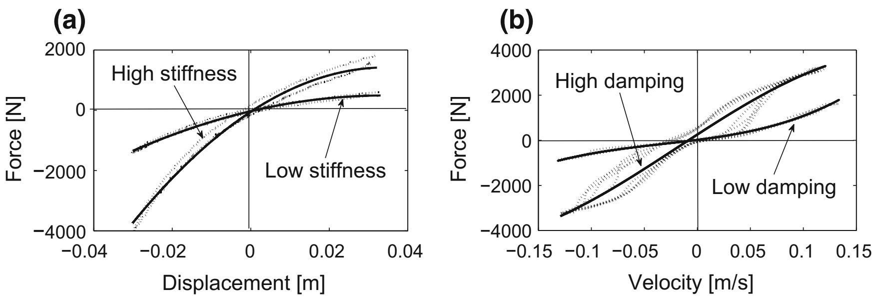

Fig. 1.4 Exemplary stiffness characteristic of the pneumatic spring: measurement (······), approximation (——) and damping characteristics of the hydraulic shock-absorber: measurement (······), approximation (——) [9]

There are no effective methods for shaping the vibro-isolation properties of the vibration reduction systems having non-linear visco-elastic characteristics which should be chosen for specific spectral classes of the excitation signals. A vibration isolator should be designed in such a way that efficiently reduces the vibrations in whole frequency range of the excitation signal (most often random excitation). Unfortunately, the optimisation of vibration isolators is very rarely used during a design process because the ongoing works have difficulties connected with the computational problems [10]. The low efficiency of the optimisation procedure is related to a significant number of the decision variables and problems concerning the local extrema. Problems connected with finding the global minimum of objective function occur very often in the case of design optimisation of many mechatronic systems [11]. Moreover, the choice of reliable and efficient optimisation criteria causes many difficulties for the designers. Some problems concerning the optimisation of pneumatic vibration isolators for various optimisation criteria are shown in the paper [12]. There is a lack of effective procedures of formulating and solving the optimisation problems for vibration reduction systems that are described by non-linear equations of motion. In addition, it is not known how to formulate a global objective function and which of the optimisation methods is effective for a given problem [13].

The second group of vibration reduction methods comprises the active techniques where the structural and parametric modifications are applied with the use of external power supply [14]. The actuator used in the system may generate a force that compensates the vibration coming from the exciting forces (active systems) or may change the system parameters (semi-active systems). The models of such vibration reduction systems are shown in Fig. 1.5.

The active vibration reduction systems generate the additional forces or controlled displacements which are operating directly on the isolated body so that to compensate the exciting forces or kinematic excitations (Fig. 1.5c). Unfortunately, the main drawback of such systems is the fact that they require a high amount of the power supply, and therefore, the active vibration isolators are hardly ever used in practice. Semi-active systems consist of passive elements (springs and dampers), although their force characteristics can be changed very fast (Fig. 1.5a–b). Such systems do not require strong power supply. Thus, semi-active vibration isolators are very often used in practice [15, 16].

The main problem of the semi-active or active vibration isolators concerns the selection of control strategy. Classical methods of vibration control significantly improve the vibro-isolation properties of the linear systems while the system parameters are exactly known. Unfortunately, the systems discussed in the literature are based on the simplified linear models that result in significant discrepancies compared with the real vibration isolators. In reality, the system parameters vary in a wide range and this leads to the lower effectiveness of active vibration isolator. Therefore, the parameter variations have to be taken into account especially using simplified models describing the vibration reduction systems. In the paper [17], the robust control system of active car suspension is presented that takes into account the uncertainties of visco-elastic elements used in the system. Successively in the paper [18], the robust control strategy is applied to vibration control of a rotor that

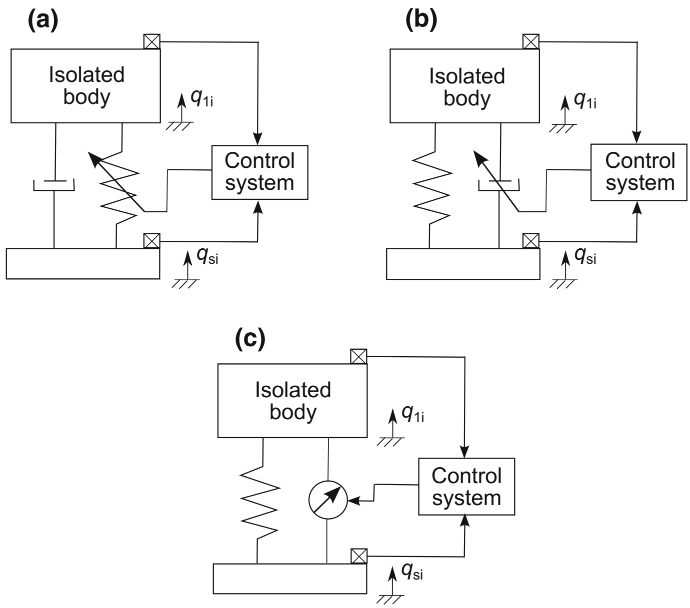

Fig. 1.5 Models of the vibration reduction systems: semi-active system with spring control (**a**), semi-active system with damper control (**b**), active system (**c**)

is suspended using the magnetic bearings with non-linear characteristics. In many papers [19–21], the modern vibration isolators are proposed and these systems are still being developed in terms of the effective control strategy.

In the recent literature, there is no systematic knowledge concerning the procedures to be followed in the design process of semi-active or active vibration reduction systems. The control system synthesis, especially if this synthesis is based on non-linear models, is difficult, when the semi-active or active system has to restrict conflicted vibro-isolation criteria. It is also not clearly defined, which of the controllers [22–25] can be applied in modern vibration isolator and how their controller settings should be calculated. Summarising, there is no effective method to define the control system structure and optimise the controller settings that can be utilised for the designed semi-active or active vibration reduction system.

1.2 Research Methodology

The fundamental method of evaluating the effectiveness of vibration reduction system is to perform an experiment in the laboratory. The tested system should be excited by the signals that are representative of the different types of working machines [26–28]. The vibro-isolation criteria of the tested system can be calculated on the

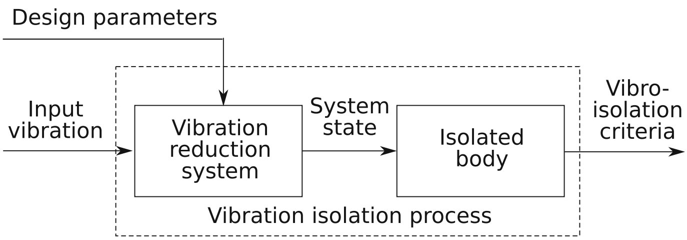

Fig. 1.6 Concept model of the vibro-isolation process

basis of measured signals. However, in many cases the technological restrictions and the high cost of the experiment make the test difficult to perform. The duration of the test is also an important aspect, especially when the test must be performed repeatedly for different design parameters of the system. Taking into account the complexity of the research process, the authors of this book recommend to carry out a simulation experiment based on a mathematical model of vibration isolation system that is shown in Fig. 1.6.

In order to analyse the dynamics of such system (Fig. 1.6), at first a theoretical model of the excitation signal with specific spectral characteristics should be elaborated. Subsequently, in order to determine the values of vibro-isolation criteria, which are very often defined as the integral values [14, 15], models of the vibration reduction system and the isolated body have to be created. For the purpose of this book, the dynamic behaviour of the vibration reduction systems is modelled in the MATLAB-Simulink® software package. The equations of motion are programmed using the interactive graphical environment, which allows to simulate and test a variety of the time-varying system.

The research presented in the papers [3, 6, 7] clearly shows that the passive vibration isolation methods in many cases prove to be ineffective. Although they provide energy dissipation at the sufficiently high frequencies, the low-frequency vibrations are amplified due to the resonance effect. In the consequence of this undesirable effect, it is difficult to achieve the desired system properties in order to meet the conflicted requirements for modern vibration reduction systems. The semi-active or active systems should improve the effectiveness of vibration isolators, and their concept model is shown in Fig. 1.7.

The feedback control of the vibration reduction system is intended to improve the dynamics of isolated body and shape the system characteristics especially for a specific vibro-isolation process realised by the system [2]. The control element in semi-active and active vibration isolation systems is adjusted by means of the controller which uses information concerning the system state and coming from a measurement system [1, 6]. However, the effective control system should allow to shape vibro-isolation properties of the system for the specified excitation signals and different working conditions. Using a unique vibration control system, whose structure and individual components will be proposed in this book, it will be possible

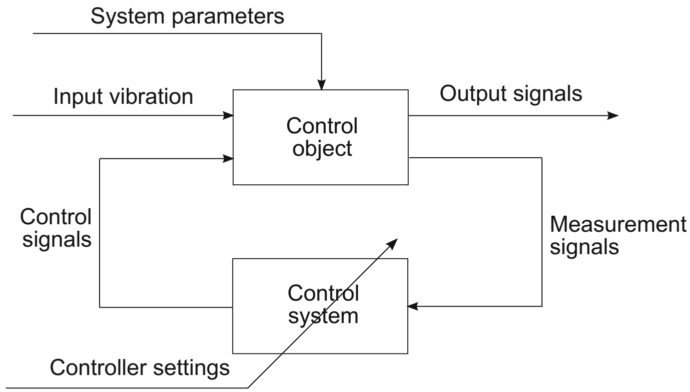

Fig. 1.7 Concept model of the semi-active or active vibration isolation system

to achieve the desired system properties in view of the conflicted requirements for modern vibration reduction systems.

The optimisation procedure proposed in this book ensures finding a set of the compromise solutions (Pareto-optimal solutions). This procedure allows to adjust the vibro-isolation properties of vibration isolator for the individual requirements defined by the machine operator. The required configurability of passive systems will be obtained by properly selecting their non-linear visco-elastic characteristics or by suitable changing of the controller settings (decision variables) in semi-active and active systems.

A selection of the vibro-isolation properties for the exemplary systems has to be performed within the framework of this work. Such a selection of the dynamic characteristics is conducted for the vibration isolation systems of different designs that are generated using the signals representing the work of different machinery. The optimisation process is led using the randomly starting points, because such procedure ensures a high probability that the optimum found is a global one. Initially, the separate minimising of the conflicted vibro-isolation criteria is carried out, and then, a minimisation of the primary criterion is recommended taking into account the other optimisation criteria.

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Chapter 2
Simulated Input Vibration

2.1 Nomenclature

$D_{\mathrm{si}}(s)$	denominator polynomials of a total filter
$G_{\mathrm{si}}(s)$	transfer function of a total filter
$G_{\mathrm{HPij}}(s)$	transfer function of the Butterworth high-pass filter
$G_{\mathrm{LPij}}(s)$	transfer function of the Butterworth low-pass filter
$\mathrm{LSE}_{\mathrm{si}}$	least square error of a specified frequency response
$N_{\mathrm{si}}(s)$	numerator polynomials of a total filter
$\mathrm{PSD}_{\mathrm{si}}$	desired power spectral density, $(\mathrm{m/s^2})^2/\mathrm{Hz}$
$\mathrm{P\hat{S}D}_{\mathrm{si}}$	estimated power spectral density, $(\mathrm{m/s^2})^2/\mathrm{Hz}$
$\mathrm{P\bar{S}D}_{\mathrm{si}}$	mean value of the desired power spectral density, $(\mathrm{m/s^2})^2/\mathrm{Hz}$
R_{si}	multiple correlation coefficient
$a_1, \ldots, a_{\mathrm{n}}$	coefficients of the Butterworth filter
f	frequency, Hz
f_{HPij}	cut-off frequency of the high-pass filter, Hz
f_{LPij}	cut-off frequency of the low-pass filter, Hz
i	directions of the vibration exposure (x, y, z)
j	number of the signal generator (1, ...,l)
k_{ij}	gain factor of the required magnitude of the power spectral density function in a specified frequency bandwidth, $(\mathrm{m/s^2})^2/\mathrm{Hz}$
n	filter order
$p(\ddot{q}_{\mathrm{ij}}(t))$	probability density function of the acceleration signal
$\ddot{q}_{\mathrm{ij}}(t)$	acceleration of the input vibration, $\mathrm{m/s^2}$
$(\ddot{q}_{\mathrm{si}})_{\mathrm{RMS}}$	root mean square value of the acceleration signal, $\mathrm{m/s^2}$
s	Laplace variable
t_{k}	computation time, s
t_{s}	time interval between samples, s
σ_{ij}^2	variance of the random numbers.

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