

Dynamical Modeling and Analysis of Epidemics

edited by
Zhien Ma & Jia Li

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Preface

The spread of infectious diseases has always been of big concerns and poses a threat to public health, as well as the economic and social developments of the human society. Thus, its prevention and control become extremely important. Quantitative studies on mechanisms of disease transmissions provide a foundation for such prevention and control, and the fundamental aim of *Epidemic Dynamics* is to investigate the transmission dynamics of infectious diseases. It formulates mathematical models, based on the occurrence and progressions of diseases and the surroundings, to characterize the infectious agents, to describe the transmission processes, to analyze origins of the diseases and factors involved in the transmissions, and to predict the prevalence of the diseases and their patterns. With deep understanding and the knowledge that we obtain from the *Epidemic Dynamics* studies, and good estimates of important factors and parameters, prevention and control strategies can be made. *Epidemic Dynamics* can employ well-developed modern dynamic theory to better characterize the inherent patterns and to investigate long-term behavior of disease transmissions. Further qualitative and quantitative studies, and sensitivity analysis on model parameters can help us to make more realistic simulations and reliable transmission prediction which may not be feasible by experiments or field studies. Moreover, the combination of epidemic dynamics, epidemiologic theory, biostatistics, and computer simulations will significantly contribute to further improvement of our knowledge of transmission patterns of epidemics, development of epidemiology, and more effective methods in controlling infectious diseases.

The Mathematical Biology group of Xi'an Jiaotong University led by Professor Zhien Ma, collaborating with Dr. Jia Li, Professor of University of Alabama in Huntsville, USA and Guest Professor of Xi'an Jiaotong University, has been working on the mathematical modeling of infectious diseases and epidemic dynamics since 1995. The Mathematical Biology group of Xi'an Jiaotong University has been assigned and achieved four important national research projects and an international collaborative

project. This book is based on what we have learned and gained throughout the course in our research.

The focus of this book is on the qualitative analysis of mathematical epidemic models. It consists of six chapters. Basic mathematical disease models, the ideas for the modeling, and fundamental concepts and techniques for the model analysis are given in Chapter 1. They are the foundations for beginners. Introductions and more advanced studies on models based on ordinary differential equations, delayed differential equations, impulse differential systems, and structured models, are provided in Chapters 2 to 5, respectively. Models on SARS (severe acute respiratory syndrome), HIV (human immunodeficiency virus) and AIDS (acquired immune deficiency syndrome), and TB (tuberculosis) are introduced and investigated to show the applications of *Epidemic Dynamics* to specific diseases in Chapter 6.

This book aims to lead the reader, who is interested in *Epidemic Dynamics*, from the fundamentals to the research frontier of the field. It provides references that the authors have collected and the results from their research projects during the past few years. The content of the book is intended to be written as easy-to-digest and comprehensive for the beginner, as well as advanced for the researcher to pursue further development. In this book, an attempt has also been made to give more rigorous descriptions of basic concepts, to emphasize modeling ideas, to introduce widely used and newly developed methods and techniques throughout well-thought and selected models, and to provide useful and detailed biological interpretations of the mathematical conclusions from the models. We believe that this book is a vital reading for applied mathematicians, public health workers, epidemiologists, and graduate students in those disciplines.

Based on planning as a whole, the book was written cooperatively as follows: Chapter 1 by Zhien Ma and Jia Li, Chapter 2 by Jianquan Li, Chapter 3 by Wendi Wang, Chapter 4 by Zhen Jin, and Chapters 5 and 6 by Yicang Zhou. Jia Li did a final edit of the book.

We would like to express our sincere appreciation to Professor Daqian Li, Member of Chinese Academy of Sciences, for his support and Professor Zhongqing Xu for his recommendation. The work of Jia Li was partially supported by U.S. National Science Foundation grant DMS-0412386.

Considering the vast literature, to select suitable materials and references for a book devoted to *Epidemic Dynamics* is a very difficult task. It is certain that many important parts have been missing or omitted, for that we bear full responsibility.

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