

Matti Leppäranta

FREEZING OF LAKES AND THE EVOLUTION OF THEIR ICE COVER

SECOND EDITION



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Matti Leppäranta

Freezing of Lakes and the Evolution of Their Ice Cover

Second Edition

 Springer

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Preface to the Second Edition

This second edition has been prepared eight years after the first one. There has been much progress in lake ice research during this period, and, consequently, a revision is well motivated. This edition contains new findings especially for low-latitude lakes in Central Asian arid climate, physical limnology below ice, remote sensing, and lake ice climatology including climate change impact on lake ice. In addition, based on comments from several readers and students of the first edition, modifications, restructuring, additions, and technical improvements have been made in most sections.

The recent progress in the research of lake ice has been largely connected to ice–water interaction, lake ice climatology, and the role of ice cover in lake ecology. Remote sensing has also much progressed together with increased availability of high-resolution images. The cryosphere has become more important in the Earth system science, and lake ice forms one specific part of the cryosphere with major local environmental and practical impacts.

The author is grateful to his collaborators in the research of lake ice science. The author is especially thankful to the participants of Winter Limnology field courses in Lammi Biological Station, Finland, and Dalian University of Technology, as well as lecture courses in Saint Petersburg State University and Institute of Northern Water Problems in Petrozavodsk. Fruitful contacts have continued with the majority of the colleagues listed in the preface of the first edition. Others to thank are professors Yila Bai, Lars Bengtsson, Irina Fedorova, Ivan Mammarella, Yubao Qiu, Xiaohong Shi, Jia Wang, Lijuan Wen, and Johny Wüest, and Drs. Xiao Deng, Wenfeng Huang, Johanna Korhonen, Peng Lu, and Elena Shornikova. The list of my students in lake ice has increased by Joonatan Ala-könni, Xiaowei Cao, Salla Kuittinen, Marjan Marbouti, Shuang Song, Dongshen Su, Fang Yang, and Yaodan Zhang.

The author also wants to thank the Springer team for good collaboration. Support to continue lake ice research has been available from the Academy of Finland, Chinese Academy of Sciences, and European Union 7th Framework Programme.

Lahti, Finland
February 2022

Emeritus Professor Matti Leppäranta

Preface to the First Edition

Ice cover is an essential element in cold climate lakes. To a large degree, it isolates the water body from the atmosphere and sunlight and slows down physical and biological processes. In the boreal zone, tundra, and mountain regions, the ice cover is seasonal, while in very high altitudes and high polar latitudes, perennially ice-covered lakes are found. Ice formation results in different ice types and stratigraphy, which further influence on the ice properties. Ice sheets of freshwater lakes are poor in impurities. However, they may possess liquid layers or inclusions, which serve as habitats of biota. Freezing of lakes also brings both practical advantages and problems to human living conditions in cold regions. It is of great interest to know the physics and ecology of lake ice, to monitor and predict ice conditions in lakes, and to evaluate what can be the impact of climate changes to lake ice seasons.

The present book, *Freezing of Lakes and the Evolution of Their Ice Cover*, provides the status of knowledge in the physics of lake ice and the interactions between the ice cover and the liquid water body underneath. Historical developments in lake ice research are also discussed. Chapter 1 gives a brief overview and presents the research fields. The second chapter contains the classification of ice-covered lakes, and in Chapter 3, the structure and properties of lake ice are presented. Ice growth and melting are treated in Chapter 4, while the following chapter focuses on ice mechanics. Chapter 6 goes into the more exotic environment of pro-glacial lakes. The last three chapters consider important lake topics related to the presence of ice in lakes. Chapter 7 contains the physics of the water body beneath lake ice. Chapter 8 discusses the winter ecology of freezing lakes and the lake ice interface toward the society including the impact of climate change on lake ice seasons. The book ends into a brief closing chapter and list of references. Examples of research problems for student learning are listed throughout the book.

The underlying idea behind the book has been to include the whole story of lake ice into a single volume. There is a crying need for such synthesis, as winter limnology research and applications are increasing and, apart from review papers, no comprehensive monograph exists on this topic in English. The author has contributed to lake ice research since the 1980s. In particular, his topics of interest have been lake ice structure and thermodynamics, light transfer in ice and snow, ice mechanics

in large lakes, and lake ice climatology. Mathematical modeling of ice growth, drift, and decay are covered in this research.

This book has grown from the author's research, collaborative visits to several universities and research institutions, and intensive international courses—winter schools—in lake ice and winter limnology. These visits concern in particular the Estonian Marine Institute of Tartu University (Tallinn, Estonia), Institute of Limnology of the Russian Academy of Sciences (Sankt Petersburg, Russia), Institute of Low Temperature Science of Hokkaido University (Sapporo, Japan), Leibniz-Institute of Freshwater Ecology and Inland Fisheries (Berlin, Germany), Marine Systems Institute of Tallinn University of Technology (Tallinn, Estonia), Northern Water Problems Institute of the Russian Academy of Sciences (Petrozavodsk, Russia), and the State Key Laboratory of Coastal and Offshore Engineering of Dalian University of Technology (Dalian, China). The winter schools have been arranged in Lammi Biological Station and Kilpisjärvi Biological Station of the University of Helsinki, in the Saroma-ko field site of the Hokkaido University, and in the Dalian University of Technology. Very recently, the author joined the Global Ice Modeling Project of the Global Lake Ecological Observatory Network (GLEON).

In the progress of his research, the author has learned about lake ice from a large number of colleagues. Especially, he wants to thank Prof. Erkki Palosuo as well as Professors Lauri Arvola, Nikolai N. Filatov, Robert V. Goldstein, Hardy B. Granberg, Timo Huttula, Toshiyuki Kawamura, John E. Lewis, Zhijun Li, Anu Reinart, Kalevi Salonen, Matti Tikkanen, Kunio Shirasawa and Juhani Virta, and Drs. Helgi Arst, Cheng Bin, Christof Engelhardt, Ants Erm, Glen George, Sergey Golosov, Sergey Karetnikov, Georgiy Kirillin, Esko Kuusisto, Nikolai M. Osipenko, William Rizk, and Arkady Terzhevik. He is also deeply thankful to his present and former students, in particular Ms. Elina Jaatinen, Mr. Juho Jakkila, Dr. Onni Järvinen, Ms. Anni Jokiniemi, Mr. Tom Kokkonen, Dr. Ruibo Lei, Ms. Elisa Lindgren, Ms. Shi Liqiong, Ms. Ioanna Merkouriadi, Dr. Jari Uusikivi, Dr. Caixing Wang, Dr. Keguang Wang, and Dr. Yu Yang. The scientists and technicians, who have participated in lake ice research programs from this institute and from several other organizations, are gratefully acknowledged, as well as students of the summer and winter schools. Ms. Elisa Lindgren is thanked for a very careful review of the manuscript, and Dr. Salla Jokela is thanked for several graphic products for this book.

The home institute of the author since 1992, the Department of Geophysics (fused to Department of Physics in 2001) of the University of Helsinki, is deeply thanked for good working conditions and support. This work is a contribution of the Nordic Center of Excellence CRAICC (Cryosphere–atmosphere interactions in a changing Arctic climate), and the results are based on several research projects, primarily Ficca and Finnarp programs of the Academy of Finland and Clime project of the EU V Framework. The preparation of this book was made possible by financial support for a sabbatical year from the Finnish Cultural Foundation, which is gratefully acknowledged.

Helsinki, Finland
August 2014

Matti Leppäranta

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Chapter 1

Introduction



At a hole in lake ice, a painting from the year 1900 by Finnish artist Pekka Halonen (1865–1933), who is well known especially of his winter landscapes. In cold regions the normal life has adapted to the presence of frozen lakes. Holes in ice cover have been used for household water, flushing clothes, and cold bathing. © *Finnish National Gallery*, printed by permission

A lake is a liquid water body in a depression on the surface of the Earth. It is associated with a water balance, which largely determines the lake volume and the water quality. The size of lakes ranges from small ponds to large intra-continental basins with an area more than 10^4 km². The global water storage of lakes is 125,000 km³, and about 80% of that is fresh water,¹ accounting for the fraction of 0.3% of the Earth's freshwater resources (Henderson & Henderson, 2009). In the boreal zone, tundra, Central Asian cold and arid climate zone, and high mountain regions, lakes freeze over in winter. The ice season may last more than half a year, and the thickness of the ice can reach two meters. In high polar zones and high mountains, there are perennially ice-covered lakes, with ice cover several meters thick. Sub-glacial lakes are located at the base of the 4 km thick Antarctic ice sheet and represent an extreme case of lakes under a very thick ice cap on Earth.

1.1 History

Frozen lakes have belonged to people's normal life in the cold regions. Solid ice cover has been an excellent base for traffic across lakes, to travel, and to transport cargo, provided the ice has been thick enough (Fig. 1.1). Indeed, an ice cover isolates island residents from the mainland for periods when the ice is too weak to walk but still limits boating. These periods bear in Finnish language an old word *kelirikko* (from 'keli' = traffic conditions, and 'rikko' = broken); in Swedish this word is *menföre*, or in Russian it is '*pacнymуя*' (rasputica). One of the first records of lake ice events is '*The battle on the ice*', a winter clash between the forces of the Norwegian king Åli and the Swedish king Aðals on the ice of Lake Vänern in Sweden, at about year 530 AD (Osborn & Mitchell, 2007). The legend of the battle on ice of Lake Peipsi in February 1246 was filmed by Sergey Eisenstein in his movie *Alexander Nevsky* (1938). The chart *Carta Marina* of Olaus Magnus Gothus (1539) over Northern Europe and his book on northern peoples (1555) showed drawings and descriptions of crossing over ice-covered lakes by skis, skates and horses and reported of horse races on ice in the Middle Ages. During the Little Ice Age (1450–1850), skating and games on lake ice spread all over to Central Europe.

Winter roads on lake ice and river ice together with snow-covered overland sections formed a very important traffic network in northern Eurasia in the prehistoric times. These winter networks provided easy ways of crossing between hydrological drainage basins that is a strong limitation in boat traffic. Winter roads on lake ice are still in use, kept by road maintenance authorities as well as by private people and industry. Also, lake ice serves as an excellent landing platform for aircraft. In wood industry, logs have been stored in winter on ice to be dragged or floated along

¹ Fresh water refers to natural water with very low concentration of dissolved substances. There is no strict limit but the concentration 0.5‰ (or 0.5 g L⁻¹, L denotes litre) serves as a convenient reference. This limit is also often used for the upper limit of the salinity of drinking water.



Fig. 1.1 Lake Äkäslompolo in southern Lapland in winter. *Photograph by the author*

streams to factories after ice melt. Until about mid-twentieth century, when refrigerators were not common, the cold content of lake ice was utilized. Ice blocks were cut in winter, stored under sawdust or hay to protect from melting, and in summer used as a cooler for foods, especially in dairy farms. Openings are cut by ice-saws into lake ice covers for household water and cold bathing, which is in modern times claimed to be a healthy habit although in the Middle Ages these baths were used as a punishment. For ice-covered lakes, special techniques have been developed for domestic and commercial winter fishing. Recently, recreation activities have become important utilizers of frozen lakes, particularly for nature tourism, sport fishing, long-distance skating, skiing, ice sailing and many kinds of games. There, safety questions are a major issue.

1.2 Lake Ice Science

The Earth's surface waters are classified into lakes, rivers, and seas. In cold regions, ice occurs in these basins as lake ice, river ice, sea ice, and blocks of land ice² forming the category of *floating ice*. Rivers have a dynamic ice cover due to their permanent and turbulent flow, resulting in abundance of frazil ice, anchor ice and ice jams. Sea ice forms of saline water and contains brine, and large sea ice basins are governed by

² Land ice is of glacial origin, brought to water bodies by calving of glaciers.

the presence of drift ice. Massive icebergs belong to the polar ocean environment, but small blocks of land ice can be found in proglacial lakes.

Lake ice is the simplest form of floating ice because of the weaker role of dynamics as compared with sea ice and river ice. The research history of lake ice shows gains from sea ice and river ice research, where practical problems have often led to earlier innovations. For the background references, the status of sea ice physics is presented in Leppäranta (2011), Thomas (2016), Wadhams (2000) and Weeks (2010), and for river ice a good coverage is contained in Ashton (1986), Ferrick and Prowse (2002), Hicks (2016) and Shen (2006). In saline lakes the small-scale structure of ice resembles sea ice, and in very large lakes drift ice occurs as in marine basins. In freezing lakes with a notable and permanent throughflow, ice features such as in freezing rivers can be found.

Scientific research on lake ice was commenced in the 1800s along with the birth of physical limnology and lake hydrology. Scientists in the European Alpine countries played a strong role in early physical limnology, and therefore it is natural that lake ice was an important topic from the beginning (e.g., Götzinger, 1909; Verescagin, 1925). Lake ice monitoring was commenced in northern Europe in the 1800s along with the rise of geosciences (Nordqvist, 1887; Simojoki, 1978). The ice season modifies the hydrological year in cold regions by its influence on the annual distribution of runoff and by the problems caused by ice to the management of water resources. Ice engineering problems were included in scientific research from the late 1800s (Barnes, 1928), concerning shipping, ice forces on structures, and the bearing capacity of ice.

Lake ice phenology³ has been a major research topic, since ice seasons show large variability, which is a critical issue to the local population (Simojoki, 1940). Collection of ice phenology time-series of inland waters was commenced in the 1800s (Hällström, 1839; Levänen, 1894), and now they are of great importance in the research of the climate history. For example, nearly 200 years long lake ice time series are available from 1833 for Lake Kallavesi, Finland (Simojoki, 1959), and from 1850 for Lake Mendota, Wisconsin (Wing, 1943). Even longer series exist: e.g., from 1443 for Lake Suwa, Japan (Arakawa, 1954), and from 1696 for River Tornionjoki, Finland (Kajander, 1993). Records of lake ice winters in Central Europe have been used to examine the occurrence of extreme cold winters (Maurer, 1924; Takács et al., 2018). Winter temperature conditions under ice cover were also investigated in the early lake research (Homén, 1903), and ecology of frozen lakes gained more attention only later (see, e.g., Greenbank, 1945; Hutchinson, 1957; Vanderploeg et al., 1992).

Research of ice-covered lakes remained for long, until about 1970s, scattered and occasional. The annual cycle of lakes was taken into the topics of the International Hydrological Decade 1965–1974 with ice season then considered in cold regions (Falkenmark, 1973). Thereafter, more effort was made on polar lakes as the access to reach them was better and automated measurement techniques had become feasible (Vincent & Laybourn-Parry, 2008). After the year 2000, lake ice science has regained much attention, due to environmental problems and due to climate change

³ Phenology refers to studies of the dates of recurring phenomena; it originates from the Greek word φαίνω (*phainō*), ‘to make appear’.

impact questions (Bengtsson, 2011; Duan & Xiao, 2015; George, 2009; Karetnikov et al., 2017; Kirillin et al., 2012; Leppäranta, 2014; Magnuson et al., 2000). Satellite remote sensing of lake ice has progressed due to the availability of high-resolution imagery (Duguay et al., 2015; Qiu et al., 2019).

Winter biology of lakes has been an increasing research field (Melnik et al., 2008; Salonen et al., 2009; Shuter et al., 2012; Song et al., 2019). Oxygen is a major question since its renewal is limited in the presence of ice cover (Golosov et al., 2007; Huang et al., 2021). Questions of life in extreme conditions have brought in more research of lakes with perennial ice and pro-glacial lakes (Keskitalo et al., 2013; Leppäranta et al., 2020; Priscu, 1998; Vincent & Laybourn-Parry, 2008), especially in the Antarctic continent. The ecosystem of subglacial lakes has been largely unknown, but the first samples ever taken from these lakes showed signs of life, mainly bacteria (Christner et al., 2006; Rogers et al., 2013).

Presently the leading research topics are the light transfer through ice, interaction between the ice and the water body, remote sensing, winter ecology, and lake ice climatology. Characteristics of lake ice seasons are quite sensitive to climate variations. For human living conditions, shorter ice seasons, while extending the open water season, with thinner ice would severely or even drastically limit traditional on-ice activities. Climate change issues also necessitate a better understanding of the role played by lake ice cover in the emission of greenhouse gases, especially methane, into the atmosphere, and in the global carbon budget.

Mathematical modelling applications in lake ice research have concerned ice growth and decay, radiation transfer, ice forces, and ice drift or ice displacements. The models are used for basic science, forecasting, ice engineering, environmental research and protection, and lake ice climate research. Two classical, analytical models, which are still applicable as the first approximations, are the ice growth model by Stefan (1891) and the bearing capacity model by Hertz (1884). The former model predicts ice thickness to be proportional to the square root of the freezing-degree-days, and the latter model predicts the bearing capacity for a point load to be proportional to the squared ice thickness (Fig. 1.2). Also, semi-empirical ice phenology models were used in the past. Prediction of the freezing date was based on the weighted integral of the air temperature history (Rodhe, 1952), while ice breakup-date was modelled based on the integral of positive degree-days (Williams, 1971). More advanced thermodynamic approach to ice season was given by Pivovarov (1973).

Until 1990s, thermodynamic lake ice models were mostly semi-analytic, based on the freezing-degree-days for ice growth and positive degree-days for melting (e.g., Ashton, 1986; Leppäranta, 1993, 2009). Also, numerical models were employed for the evolution of ice thickness and temperature (e.g., Croley & Assel, 1994; Leppäranta, 1983; Stepanenko et al., 2019; Yang et al., 2012). Numerical sea ice models had been developed earlier (Maykut & Untersteiner, 1971; Saloranta, 2000; Semtner, 1976) and they were utilized also for lake ice. The snow layer, which plays an important, interactive role in the evolution of the thickness of ice, especially in the boreal zone, forms the most difficult part of lake ice thermodynamic models.



Fig. 1.2 The bearing capacity of lake ice is a major safety question for on-ice traffic. The photograph shows a rescue operation of a car in Lake Päijänne, Finland in spring 2005. Published in newspaper *Etelä-Suomen Sanomat* on March 31, 2005. Photograph by Mr. Pertti Louhelainen. Printed by permission

Mechanical models have been developed for ice forces in the scales of man-made structures (Korzhavin, 1962; Michel, 1978; Palmer & Croasdale, 2012) and for drift ice for whole lake basins (Leppäranta & Wang, 2008; Ovsienko, 1976; Wake & Rumer, 1983; Wang et al., 2005). The drift of ice is forced by winds and currents, and the mobility of ice is determined by its strength and the size of the basin. Ice drifts in very large lakes, such as the Caspian Sea and Lake Superior, as in marine basins, and in this field as well sea ice models were the basis of lake ice models (Coon, 1980; Doronin, 1970; Hibler, 1979; Wang et al., 2003; Wang et al., 2010).

1.3 Ice Season

By far, most freezing lakes have a seasonal ice cover. The ice is normally thin as compared with the lake depth. The physical properties of ice show large random variability due to the high homologous temperature,⁴ the large size of individual crystals, and the presence of impurities. Lake ice shows a simple stratification pattern consisting of congelation ice formed of the lake water and snow-ice formed of eventual slush on ice (e.g., Adams & Roulet, 1980; Leppäranta & Kosloff, 2000; Michel &

⁴ The temperature of a medium as a fraction of its melting point temperature (Kelvin scale).

Ramseier, 1971; Palosuo, 1965). Snow accumulation on lake ice is common apart from cold and arid climate regions (e.g., Huang et al., 2012; Obryk et al., 2014). Even a thin layer of snow significantly weakens the heat exchange between the lake water body and the atmosphere, and the transfer of sunlight into the water. Frazil ice formation may occur in the first stage of freezing lakes, but later on, apart from large lakes, it seems to be rare.

The evolution of a lake ice season is primarily a thermodynamic process (see Kirillin et al., 2012; Leppäranta & Wang, 2008; Michel & Ramseier, 1971). After the freeze-up, when the ice is thick enough, it spans a stable lid over the lake. Ice breakage and mechanical displacements may take place all winter in large lakes, but in smaller lakes only if the ice is thin or weak. Decay of a lake ice cover is a thermo-mechanical process, starting from the shoreline (Fig. 1.3). Melting of near-shore ice releases the ice from solid boundaries, and the ice cover may then shift as forced by winds and currents. Ice loses its strength and bearing capacity due to internal deterioration. Movement of ice causes further breakage of ice that speeds the decay process.

Land-ice interaction due to thermal expansion and onshore ride-up or piling-up is an important environmental and practical issue. It has attracted geographers for more than one hundred years (Alestalo & Häikiö, 1979; Buckley, 1900; Helaakoski, 1912). Due to thermal and mechanical stresses, lake ice deforms shore areas and loads man-made structures at the shoreline. In shallow areas, bottom scouring by ice and freezing through the whole water column give rise to erosion of the lake bottom. Land-ice interaction has therefore geological and biological consequences.



Fig. 1.3 Ice break-up takes place in spring in boreal lakes. During a few weeks prior to the disappearance of ice, the ice cover is weak and patchy and impossible to cross by foot or boat. Lake Päijänne, Finland, end of March 2017. *Photograph* by the author

In a lake water body, physical phenomena and processes are very different under ice cover from the open water conditions. The ice cover cuts the transfer of momentum from the wind to the water body that damps turbulence and mixing. We may say that the time slows down. The surface water temperature is at the freezing point, there is very little vertical transfer of heat, apart from geothermal lakes, and only a weak thermohaline circulation is present. The temperature structure and circulation are quite stable, and salinity stratification may be important even in freshwater lakes. In large lakes, the ice cover may experience episodic movements and disturb the water body. When available, solar radiation provides strong heating, in fact the strongest forcing given into fully ice-covered lakes, resulting in convective deepening of the mixed layer (Kirillin et al., 2012). During ice melting, the meltwater with its impurities is released into the water body.

The volumetric changes in the liquid water body, associated with the formation and growth of ice are of minor consequence in deep lakes, but in very shallow lakes, substantial fractional reductions in the volume of liquid water may result that also effects the geochemistry of the lake water (Yang et al., 2016). In shallow lakes the bottom sediments have an important role in the heat budget. They collect large heat storage in summer and release the heat back to the water body in winter. Gas release from sediments forces vertical circulation, and impurities may dissolve from the sediments to enrich the near-bottom waters and create a thin bottom boundary layer from the resulting salinity stratification (Golosov et al., 2007; Lu et al., 2020). This near-bottom layer also has impact on the near-bottom living conditions.

Lake ice formation and growth have influence on the water quality and on the lake ecology (e.g., Kirillin et al., 2012; Salonen et al., 2009; Song et al., 2019). The concentration of suspended matter decreases, since due to weak inflow and turbulence, sedimentation is not compensated by resuspension. At the time of ice breakup, the load of impurities peaks from the meltwater of the drainage basin. A critical factor is that the ice cover reduces the level of dissolved oxygen in the water body by cutting the influx from the atmosphere (Bengtsson, 2011; Golosov et al., 2007; Greenbank, 1945; Hargrave, 1969; Huang et al., 2021). At the beginning of the ice season, the oxygen content depends on the strength of the autumn mixing. In windy autumn, the whole water body may cool down even to 1 °C, and the cold water will then contain a large amount of oxygen. In winter, oxygen is consumed mostly at the lake bottom by bacterioplankton through organic matter mineralization. This results in long winters in anoxic conditions in the deep water and, as a consequence, fish kills (Fig. 1.4).

Sunlight is the main limitation for primary production under ice cover. Nutrient levels are usually sufficient after the autumn mixing. Photosynthesis is paused for the polar night, and in the case of a thick snow cover on ice, the dark season is longer and extends to sub-polar latitudes. In the Central Asian arid climate, freezing lakes are snow-free and photosynthesis continues over all winter under ice (Song et al., 2019). In general, vertical stratification with a thin upper layer can be favourable for primary production beneath a bare ice cover, and an under-ice bloom may form if the bare ice period lasts long enough.



Fig. 1.4 Fish kill in Lake Äimäjärvi, southern Finland, March 2003. Ice season 2002–2003 began exceptionally early (late October), and the oxygen storage was not sufficient for the whole ice season. *Photograph by Mr. Jouni Tulonen, printed with permission*

1.4 This Book

The second edition of *Freezing of lakes and the evolution of their ice cover*, presents an update (year 2022) to the first edition (2014) of the status of knowledge of the physics of lake ice with applications (Fig. 1.5). The text has been prepared eight years after the first one providing new material especially for low latitude lakes in Central Asian arid climate, physical limnology below ice, remote sensing, and lake ice climatology including climate change impact on lake ice. In addition, modifications, restructuring, additions, and technical improvements have been made in most sections. Recent progress in the research of lake ice has been largely connected to ice–water interaction, lake ice climatology, and the role of ice cover in lake ecology. Remote sensing has also much progressed together with increased availability of high-resolution images.

The book has focus on freshwater lakes, with river ice and sea ice results utilized where necessary. The role of dissolved substances of the parent water is explained here based on results from sea-ice research, since research ice in saline lakes is still quite limited. A historical view is embedded with material from the more than 100 years long period in lake ice research. Earlier literature of the physics of ice-covered lakes is quite sparse. The key lake ice books include Ashton (1986), Barnes



Fig. 1.5 Learning of frozen lakes in the field is an essential corner stone of knowledge. Students of winter limnology field course learning to use an underwater robot below ice cover in Lake Lovojärvi, Finland. *Photograph by the author*

(1928), Michel (1978), Pounder (1965) and Shumskii (1956). Pivovarov (1973) presented a scientific monograph on freezing lakes and rivers with weight on the liquid water body, while Ashton (1986) had a more engineering point of view on this topic [see also Ashton (1980) for a condensed review paper on the topic]. More recent material is given in the article collections by George (2009), Vincent and Laybourn-Parry (2008), and in the reviews of Kirillin et al. (2012) and Shuter et al. (2012). Recently a book about mountain lakes in the Sierra Nevada of California was prepared by Melack et al. (2021).

The first chapter introduces the topic with a brief historical overview. Chapter 2 presents lakes, their classification, the zones of ice-covered lakes, and remote sensing methods. Ice formation and the structure and properties of lake ice are treated in Chap. 3, including ice impurities and light transfer. Chapter 4 contains lake ice thermodynamics from the freezing of lakes to ice melting, with thermodynamic models included. Lake ice mechanics is the topic of Chap. 5 with engineering questions such as ice forces and bearing capacity of ice. A section is included on drift ice in large lakes. Glaciers and pro-glacial lakes are treated in Chap. 6. Quite exotic lakes are introduced, particularly lakes at the top and bottom of glaciers and ice sheets. Chapter 7 focuses on the water body beneath lake ice cover with water balance, stratification, and circulation. Ecology and environmental questions of ice-covered

lakes, and lake ice and society including practical questions are treated Chap. 8. The next chapter focuses on lake ice climatology looking into time series and analytic modelling of climate sensitivity and climate impact. The final closing words are written in Chap. 10, and the list of references follow in Chapter 11. Study problems with solutions are given as examples in the text. Properties of ice and liquid water and useful constants and formulae have been collected in the annex.

Lake ice belongs to the cryosphere part, which closely interacts with human living conditions and industry. The ice cover has caused problems, but people have learnt to live with them and to utilize the ice. Especially, in the present time this is true for on-ice traffic, recreation activities, and tourism. Ice fishing has become a widely enjoyed winter hobby. Winter sports such as skiing, skating, and ice sailing are now popular activities on frozen lakes. The interaction between lake ice and human life is to stay to the future, but modifications may be brought along the evolution of the climate. The lake ice response to climate warming would appear as the shortening of the ice season due to increasing air temperature, but also the quality of the ice and ice seasons would change via the climate influence on the thickness and structure of the ice.

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Chapter 2

Freezing of Lakes



Terra/Aqua—MODIS (Moderate Resolution Imaging Spectroradiometer) image of the eastern Great Lakes region on February 28, 2004. White ice floes seen on Lake Huron, top left, and Lake Erie, bottom centre. Deeper Lake Ontario, right, remains free of ice, though the land around it is in winter's icy grip. The mean depth and area are 19 m and 26,000 km² in Lake Erie and 86 m and 19,000 km² in Lake Ontario. NASA. Production by The Visible Earth team (<http://visibleearth.nasa.gov/>)

2.1 Lake Types and Characteristics

2.1.1 Lake Basins

Lakes are individuals. Each lake has its own geometry, bathymetry, drainage basin, and outflow. The first-order geometrical properties of lakes are based on their lateral or horizontal and vertical extent (Table 2.1). The lateral dimensions are taken as the length of the major axis ℓ , the width w , defined as the maximum length perpendicular to the major axis, and the area A . The characteristic linear dimension of the lake is taken as ℓ or $\sqrt{A}$. The largest lake is the saline Caspian Sea (370,000 km²), and the largest freshwater lake is Lake Baikal by volume (23,620 km³) and Lake Superior by area (82,100 km²). Lake Ladoga is the largest freshwater lake in Europe (17,700 km²). There is no physical lower size limit for a lake, but in limnology very small lakes, with area less than 50,000 m², are rather called ponds. The vertical dimension of a lake is taken as its mean depth $\tilde{H}$ or its maximum depth $H_{\max}$. The deepest lake is Lake Baikal, the mean depth is 760 m, and the maximum depth is 1620 m.

The five geometric dimensions $\{A, \ell, w, \tilde{H}, H_{\max}\}$ define four dimensionless characteristics: elongation γ , shape factor κ , steepness ξ , and the aspect ratio δ :

$$\gamma = \frac{\ell}{w} \geq 1, \kappa = \frac{A}{\ell w} \leq 1, \xi = \frac{H_{\max}}{\tilde{H}} \geq 1, \delta = \frac{\tilde{H}}{\ell} \quad (2.1)$$

Normally $\delta \sim 10^{-3}$ but goes up to $\delta \sim 10^{-2}$ for small lakes. The hypsographic curve $\Pi(H)$ is equal to the fractional area of the lake deeper than H :

$$\Pi(H) = \int_H^{H_{\max}} \pi(z) dz \quad (2.2)$$

where π is the spatial density of lake depth.

Example 2.1 Take a cone-shape lake with radius R and maximum depth $H_{\max}$. Then $\gamma = 1, \kappa = \frac{\pi}{4}, \xi = 3, \delta = \frac{H_{\max}}{6R} = \frac{\tilde{H}}{2R}$, and $\Pi(H) = \pi R^2 \left[1 - \left(\frac{H}{H_{\max}} \right)^2 \right]$.

Table 2.1 Classification of lakes for their lateral and vertical size

Lateral ℓ	Very large 1000 km	Large 100 km	Medium 10 km	Small 1 km	Very small 100 m
Vertical $\tilde{H}$	Very deep 500 m	Deep 100 m	Medium 20 m	Shallow 5 m	Very shallow 1 m

A single lake is a connected unit so that liquid water particles can circulate throughout. There may be sub-basins with straits between, sometimes taken as separate lakes. The geometry of lakes has fractal characteristics (Korvin, 1992). The length of the shoreline $\mathcal{L}$ increases with the measurement step Δs as a power law: $\mathcal{L} \propto (\Delta s)^{1-D}$, where $D > 1$ is the fractal dimension or the Hausdorff dimension of the shoreline; thus, $\mathcal{L} \rightarrow \infty$ as $\Delta s \rightarrow 0$. Therefore, the length of the shoreline is not a well-defined geometrical property of a lake. Similarly, the number of lakes in a region depends on the resolution of the map and makes sense only if a minimum size is defined. Instead, the surface area and the lengths of the major and minor axes of a lake are well defined. Apart from the descriptive geometry, however, the fractal concept has not brought much new applicable information to lake research. The shoreline geometry is indeed an important factor during the ice season, but the length scale of this factor is finite and process dependent and there is no direct link to the Hausdorff dimension.

A natural *geological classification* of lakes is according to the origins of the depressions, where lakes have formed (Table 2.2). The time scale of the depressions is usually very long, apart from those created by landslides and environmental engineering. The formation of glacial lakes has taken place during retreating of glaciers and ice sheets. In the Northern Europe and North America there are large lake districts, which originate from ice sheet–land processes after the Last Glacial Maximum about 25,000 years ago. Proglacial lakes receive their inflow mainly from living glaciers and ice sheets, and they are classified into epiglacial, supraglacial and subglacial lakes (Hodgson, 2012; Leppäranta et al., 2020). Supraglacial lakes are a particular category in that their base is ice rather than rock or soil. In the Antarctic dry valleys there are hypersaline lakes, such as Lake Vanda, with perennial lake ice (Gibson, 1999). Epishelf lakes are freshwater lakes dammed behind ice shelves, overlaying seawater or being connected to the sea by a conduit (Davies et al., 2017).

Example 2.2 The Baltic Sea has a dynamic history since the retreat of the Weichselian ice sheet, beginning 15,000 years ago, due to the positive freshwater balance and rebound of the ground from the pressure of the ice sheet (see Leppäranta & Myrberg, 2009). First, the basin was freshwater Baltic Ice Lake, then (11,600–10,800 ago) brackish Yoldia Sea, connected via Närke Strait to the Atlantic, and again (10,800–9000 years ago) freshwater Ancylus Lake (Fig. 2.1). About 9000 years ago, the Atlantic outflow changed to a strait in Denmark, and Ancylus Lake turned into a brackish marine basin, which has further developed to the present Baltic Sea. The land uplift is still in progress in the northern Baltic Sea, and in 1000 years the northern basin, the Bay of Bothnia, will be isolated from the oceanic connection and become the largest freshwater lake in Europe. Other large Holocene lakes in the Baltic Sea drainage basin are Ladoga, Onega, Vänern, Peipsi, and Saimaa.

Table 2.2 The origins of lake basins with examples from the cold climate zone

Class	Mechanism for depression	Example
Tectonic	Plate shifts	Lake Baikal (Russia)
Volcanic	Caldera	Lake Shikotsu (Hokkaido, Japan)
Meteorite	Impact crater	Lake Lappajärvi (Finland)
Landslide	Land mass accumulation	Attabad Lake (Pakistan)
Reservoir	Man-made	Rybinsk reservoir (Volga basin, Russia)
Glacial	Ice sheet dynamics	Lake Saimaa (Finland)
Epiglacial	Ice sheet dynamics	Lake Mandrone (Italy)
Supraglacial	Solar radiation	Lake Suvivesi (Dronning Maud Land)
Subglacial	Geothermal heat	Lake Vostok (East Antarctica)

2.1.2 Water Budget of Lakes

Lakes receive water from precipitation directly (P) and via inflow from the drainage basin (I), and they lose water by evaporation/sublimation (E) and outflow (O). The lake water balance presents the evolution of the water storage $S = A\tilde{H}$:

$$\frac{dS}{dt} = (P - E)A + I - O \quad (2.3)$$

where t is time, and A is the area of the lake. Inflow comes from rivers and brooks, over land surface and from ground, and water flows out along a river and seeps through lake bottom to ground. Condensation (deposition) of water vapour is included as negative evaporation (sublimation), but these cases are occasional, minor contributors.

A lake without outflow is closed and loses water only by evaporation or sublimation. There are bifurcation lakes with two outflow streams. Many lakes are regulated for inflow or outflow due to needs of hydropower plants and prevention of floods or drought. On a long term, e.g., over an annual cycle, the water storage is normally in equilibrium and the left-hand side of Eq. (2.3) is zero. Then the flushing time or the water renewal time of a lake provides the basic timescale of the water storage:

$$t_R = \frac{S}{I + PA} = \frac{S}{O + EA} \quad (2.4)$$

The range is from less than one month to decades for most lakes. The inflow along land surface and the groundwater flux are the most difficult parts in the water balance.

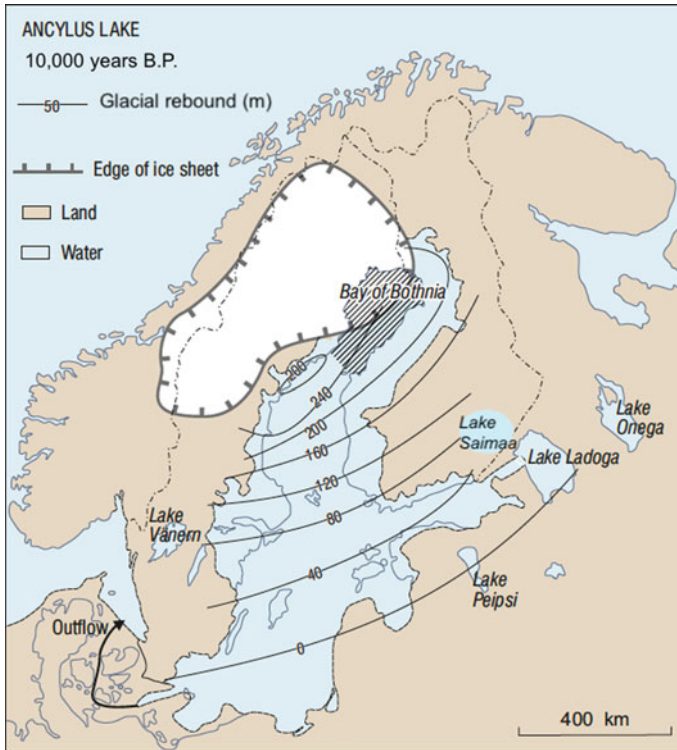


Fig. 2.1 Ancylus Lake. Also shown are the largest freshwater lake of Europe, Lake Ladoga, which seceded from the Yoldia Sea about 10,000 years ago, and the Bay of Bothnia, the future largest freshwater lake in Europe. The contour inside the Ancylus Lake shows the present Baltic Sea. Ancylus Lake has been the largest freshwater lake on Earth after the last Glacial Maximum. Modified from Leppäranta and Myrberg (2009)

Water storage can be monitored from the water level elevation. Inflow and precipitation are external factors, independent of the lake. In turn, evaporation/sublimation and outflow depend on the state of the lake, the former on the lake surface temperature and the latter on the water level elevation. Precipitation data are available from most weather stations. They are given as the accumulation rate of an equivalent liquid water layer, normally in millimetres per time. There are large difficulties to determine the solid precipitation, especially during snowstorms due to the turbulent transport of snowflakes. Therefore, recorded precipitation data from snow accumulation are often biased down. In cold climate regions precipitation gauges are equipped with specific structures to manage the snowfall aerodynamics properly (Fig. 2.2).

In cold air the saturation water vapour pressure is small, and therefore precipitation is normally low. Annual precipitation is around 500 mm in the boreal zone and half of that in tundra. Antarctic dry valleys have an extremely dry climate with no precipitation. High precipitation is recorded at windward sides of mountains, where



Fig. 2.2 Tretyakov model precipitation gauge. The metal slices around the cylinder constitute the specific design for aerodynamics to obtain representative snowfall data. *Photograph by the author*

the orography forces air masses upward. For example, in Norway, Rocky Mountains of North America, and Japanese Alps the highest annual precipitation levels are 2000–3000 mm.

Freezing and melting do not influence the overall water budget, since they just change the phase of the water molecules. When the ice is floating, liquid water level elevation is unchanged, but grounding of ice necessarily lowers the water level. Inflow is usually low during the ice season since precipitation over the drainage basin is mostly solid and forms a temporary storage on land. If the soil surface layer is frozen, the groundwater storage is blocked from top, and liquid precipitation may show up in a significant surface runoff. A low inflow lowers the water surface level, and then the natural outflow also decreases. In regulated lakes the water level is kept above a designed minimum. Closer to the climatological edge of freezing lakes, liquid precipitation events occur all winter, and the water level elevation varies more smoothly. When the melting of snow begins, the inflow starts to increase and reaches a major peak. The water level elevation has influence on the ice–land interaction, such as shore erosion and lake bottom scouring by moving ice.

Example 2.3 Specific cases of water balance.

- (a) Over the annual cycle, the net change of the water storage is normally close to zero and then $(P - E)A = I - O$. In dry regions, $P \approx 0$ and $O = 0$, and, thus, at equilibrium $AE \approx I$.

- (b) A change of the water storage, $S = A\tilde{H}$, can be expressed as the sum of the change in mean depth and the areal change: $\delta S = A\delta\tilde{H} + \tilde{H}\delta A$. For a cone-shape lake topography, the former term takes $1/3$ and the latter term takes $2/3$ of the change of the water volume.

Figure 2.3 shows an example of the annual course of the water budget in a boreal lake. In winter, the inflow was low, and the water level was slowly falling. The inflow peaked after snowmelt in April–May, that showed up delayed and smoothed in the outflow. The monthly average inflow and outflow were 610 mm and 671 mm, respectively, spread over the lake, and the flushing time was about 12 months. The excess of outflow over inflow was balanced by the positive net atmospheric flux, precipitation higher than evaporation, while the contribution of groundwater flux was at most 1% of the total. The atmospheric fluxes showed strong annual cycle, and the average precipitation and evaporation/sublimation were $56.9 \text{ mm month}^{-1}$ and $43.3 \text{ mm month}^{-1}$, respectively. The level of the estimated evaporation/sublimation was about 5 mm month^{-1} in winter and more than $100 \text{ mm month}^{-1}$ in July–August.

The quality of lake water is determined by the water balance and biogeochemical processes in the lake. Impurities are received by atmospheric deposition, resuspension of sediments, and inflow from the drainage basin. Their concentrations are changed internally by biochemical processes, and they are lost by sedimentation and outflow. The water balance Eq. (2.3) can be modified for concentrations of different

Fig. 2.3 A case of annual course of the water balance in Lake Vanajavesi, southern Finland: **a** inflow and outflow; **b** lake–atmosphere water exchange. The area of the lake is 103 km^2 , the mean depth is 7.7 m , and the area of the drainage basin is 2774 km^2 . The data are from Jokiniemi (2011) except for sublimation taken from Yang et al. (2012)

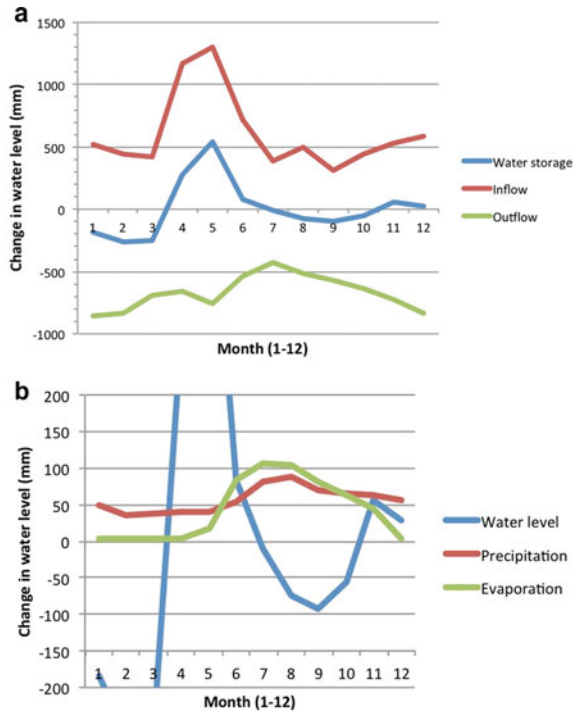


Table 2.3 Classification of lakes based on the salinity (S) of the water

Category	Salinity (‰)	Physics
Freshwater lakes	$S < 0.5$	Salinity has only marginal influence
Saline lakes • Brackish lakes	$0.5 \leq S \leq 35$ $0.5 \leq S < 24.6$	Salinity at most at the oceanic level Freezing point < temperature of max density in brackish water
Hypersaline lakes	$S > 35$	Salinity greater than the oceanic level

substances, noting that evaporation and sublimation remove only pure water. The renewal time of each substance can be defined similarly to the flushing time of the water mass. Freezing has no influence on the annual budgets of impurities, but rejection of impurities in ice growth enriches the water under ice while melting has an opposite effect. This can be a major issue in very shallow lakes (Yang et al., 2016). On the other hand, precipitation is accumulated on the ice cover during an ice season, and this storage is released to the liquid water body during the melting period, which is much shorter than the ice growth period.

In limnology, the concentration of dissolved salts is expressed as the fractional mass (salinity S) or as the mass per volume (c_d). The salinity is given in parts per thousand or per mill (‰) and the mass in grams or milligrams per liter. Clearly $c_d = \rho_w S$, where ρ_w is the density of the water sample. Lakes are classified by salinity to fresh, brackish, saline, and hypersaline types (Table 2.3). Salinity is a conservative thermodynamic state variable of lake water and plays an important role in the stratification, mixing, circulation, and freezing in saline lakes. Brackish water salinity has the temperature of maximum density above the freezing point, while in hypersaline lakes, salinities can be even above 300‰ and the freezing points is very low, even down to -50°C . The chemical composition of lake water plays an especially significant role in hypersaline lakes, for which lake specific calibration for physical properties of water would be desirable.

Saline lakes form when evaporation/sublimation plays a dominant role in the water balance, such as Lake Aral in Central Asia and Great Salt Lake in Utah. A specific class of saline lakes are tidal lakes and lagoons, which exchange water with the ocean (Dailidiene et al., 2011; Shirasawa et al., 2005). In the northern part of the Baltic Sea, due to glacial land uplift lakes becomes slowly isolated from the sea and the salinity consequently decreases (e.g., Lindholm et al., 1989). These lakes are called *fladas*.

Example 2.4 During the recent decades, the Tibetan plateau lake system has undergone major changes due to the increased air temperature and precipitation (see Qiu et al., 2019). Climate warming has brought shorter ice seasons. But increased precipitation has brought higher water volumes and therefore lower salinities in the several saline lakes in the region, and, even more, retreating glaciers are releasing more fresh-water than before that also lowers lake salinities. This desalination has increased the temperatures of the freezing point and maximum density that has impact on the freezing date in addition to the impact of increasing air temperature. What further

Table 2.4 Physical properties of pure water at the freezing point temperature and gauge pressure^a zero and the variability of these properties in cold freshwater lakes

Property	Value ($T = 0\text{ }^{\circ}\text{C}$, $p = 0$)	Variability ($T < 4\text{ }^{\circ}\text{C}$, $S < 0.5\text{‰}$)
Molecular weight	18.015 g mol^{-1}	$< 0.1\%$
Density	999.8 kg m^{-3}	$< 0.1\%$
Viscosity	$1.79 \times 10^{-3}\text{ N s m}^{-1}$	$\approx 10\%$; $1.57 \times 10^{-3}\text{ N s m}^{-1}$ at $4\text{ }^{\circ}\text{C}$
Surface tension	75.6 N m^{-1}	$< 1\%$; 75.1 N m^{-1} at $4\text{ }^{\circ}\text{C}$
Compressibility	$0.51 \times 10^{-10}\text{ Pa}^{-1}$	Very small
Speed of sound	1402 m s^{-1}	$\approx 1\%$; 1421 m s^{-1} at $4\text{ }^{\circ}\text{C}$
Specific heat	$4.22\text{ kJ kg}^{-1}\text{ }^{\circ}\text{C}^{-1}$	$< 0.1\%$
Thermal conductivity	$0.561\text{ W m}^{-1}\text{ }^{\circ}\text{C}^{-1}$	$\approx 1\%$
Latent heat of freezing	333.5 kJ kg^{-1}	–
Latent heat of evaporation	2.49 MJ kg^{-1}	$< 1\%$
Electric conductivity ^b	$< 1\text{ }\mu\text{S cm}^{-1}$	High; $\approx 100\text{ }\mu\text{S cm}^{-1}$ for $S = 0.1\text{‰}$
Relative permittivity	87.9	Sensitive to salinity

^aGauge pressure is the pressure with reference to the ambient atmospheric pressure

^bIn pure water the electric conductivity is extremely small but then rises fast with salinity

complicates the case are possible modifications in stratification, circulation, and evaporation in the lake.

The key lake water properties in ecology are pH and the concentrations of dissolved oxygen, carbon dioxide, nutrients, and harmful substances. *Ecological classification* of lake waters is based on the trophic status: the ultra-oligotrophic, oligotrophic, mesotrophic, eutrophic, and hyper-eutrophic lakes. The trophic status is related to the concentrations of the main nutrients, i.e., phosphorus and nitrogen. Consumption of oxygen is faster with higher trophic status, and therefore eutrophic lakes and more likely to reach anoxic conditions in winter.

2.1.3 Physical Properties of Lake Water

The physical properties of pure water depend weakly on the temperature (T) and pressure¹ (p) (Table 2.4). Pressure needs to be considered only in deep lakes. At the surface of high mountain lakes, the air pressure is low that has influence primarily on the saturation levels of dissolved gases. In natural waters, salinity modifies the physical properties but apart from electromagnetic properties its influence is small for fresh water ($S < 0.5\text{‰}$).

The density of lake water, $\rho_w = \rho_w(T, S, p)$ must be considered in detail, since even very small differences influence the stratification and circulation. The present

¹ Usually pressure is given in bars; the SI unit is Pascal, $1\text{ bar} = 100\text{ kPa}$.

advanced method to calculate the density is derived from the seawater standard TEOS-10 (Thermodynamic Equation of Seawater) based on the absolute salinity (see Millero et al., 2008). TEOS-10 mathematical algorithm is highly complicated and cannot be presented as an explicit formula, but there is free software available in the web for users. Since the ionic composition of water varies between lakes, in principle, each lake needs an individual equation of state (Boehrer et al., 2010). The TEOS algorithm was modified for lake water applications by Pawlowicz and Feistel (2012), and a practical solution was presented by Moreira et al. (2016).

For a general approximation, the UNESCO 1980 equation of state of sea water (UNESCO, 1981) has been employed for limnological applications. The equation of state is then expressed in explicit form as:

$$\rho_w = \rho_w(T, S, p) = \frac{\rho_w(T, 0, 0) + \Delta(T, S, 0)}{1 - \frac{p}{K(T, S, p)}} \quad (2.5)$$

where $\rho_w(T, 0, 0)$ is the density of pure water at gauge pressure zero, $\Delta(T, S, 0)$ is the salinity correction, and the denominator gives the pressure correction with the bulk modulus $K(T, S, p)$. Usually, salinity is ignored in freshwater bodies and pressure is ignored in shallow lakes. However, even in freshwater lakes, salinity may dominate the stratification below ice cover since in cold water the influence of temperature on density is minimal.

For many applications, such as first-order analysis and mathematical modelling, Eq. (2.5) is approximated by a second-order polynomial. A convenient form for cold water ($T \leq 10^\circ\text{C}$) with low salinity ($S \leq 10\text{‰}$) and gauge pressure zero reads:

$$\rho_w(T, S, 0) = \rho_w(0, 0, T_{m0}) - a(T - T_{m0})^2 + (b_0 - b_1 T)S \quad (2.6)$$

where $\rho_w(0, 0, T_{m0}) = 999.84 \text{ kg m}^{-3}$ is the reference density, $T_{m0} = 3.98^\circ\text{C}$ is the temperature of maximum density of pure water at $p = 0$, S is in ‰, and $\{a, b_0, b_1\}$ are the parameters of the formula. Taking these parameters so that the approximation is exact for pure water at the temperatures 0°C and T_{m0} , and the depression of the temperature of maximum density with salinity is properly followed, we have $a = 8.0 \times 10^{-3} \text{ kg m}^{-3} \text{ }^\circ\text{C}^{-2}$, $b_0 = 0.82 \text{ kg m}^{-3}$, $b_1 = 3.6 \times 10^{-3} \text{ kg m}^{-3} \text{ }^\circ\text{C}^{-1}$. Equation (2.6) resembles the approach for individual lakes by Moreira et al. (2016), who used lake-specific salinity parameters $\{b_0, b_1\}$ in the salinity correction.

Example 2.5 The average density of a human body is 1062 kg m^{-3} (Krzywicki & Chinn, 1966). If the water salinity is more than 75‰, the water is denser and a human body floats on the surface. There are many hypersaline lakes, which satisfy this requirement, a famous one is the Dead Sea ($S \approx 315\text{‰}$, $\rho_w \approx 1240 \text{ kg m}^{-3}$).

Freezing lakes vary widely in type (Fig. 2.4). Freshwater lakes are vertically mixed at the temperature of maximum density, and therefore the depth of a lake is one of the principal characteristics to influence the timing of the freezing. The lateral size of a lake influences the mixing conditions, since long wind fetches create